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Dynamics of fractional vortices in two-band superconductors

A. M. Kutsyk[1], A. L. Kasatkin[2], and A. A. Kordyuk[2,3]

[1]*Faculty of Radiophysics, Electronics and Computer Systems, T. Shevchenko National University of Kyiv Kyiv 03127, Ukraine*

[2]*G. V. Kurdyumov Institute for Metal Physics of NAS of Ukraine, Kyiv 03680, Ukraine*

[3]*Kyiv Academic University, Kyiv 03142, Ukraine*
E-mail: al_kas@i.ua

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The entry of fractional vortices and their subsequent dynamics inside a two-band superconductor is explored based on the numerical solutions of time-dependent Ginzburg–Landau (TDGL) equations. We consider the case when superfluid electron condensates from two zones are characterized by quite different parameters, such as coherence lengths ξ_i, and London penetration depths λ_i, which in turn leads to the different critical magnetic fields $H_{c,i}$ and fractional flux quanta ϕ_i values for the superconducting state in these two zones. Numerical solutions of TDGL equations in increasing external magnetic field followed by mathematical modeling of magnetic flux penetration were performed for this case by finite element method. We have explored the time evolution for the fractional vortices penetration process and their subsequent dynamics inside the specimens for two geometries: the circular disk, and the circular disk with a triangular cutout. Obtained results indicate that magnetic flux penetrates inside the specimen in form of fractional vortices when they can overcome the edge barrier, which may be different for these two vortex types. Therefore, in increasing external magnetic field first penetrate vortices with a lower barrier height (i.e., lower $H_{c,i}$) while the other type fractional vortices start their penetration at higher external field value. Another mechanism for the formation of fractional vortices during their entrance in a two-band superconductor is related to the difference in their flux values and viscosity coefficients which determine the rate of vortex proliferation inside the sample. Within the specimen, fractional vortices move in order to arrange. Vortices of different types attract to each other and try to stick together thus forming composite vortices with the whole flux quantum value $\phi_0 = h/2e$.

Keywords: two-band superconductor, fractional flux vortex, coherence length, edge barrier.

1. Introduction

There is significant interest in the physics of superconductors with multiple-order parameter components related to different electron zones in multiband superconductors. The latter is supported by investigations of superconducting states in new classes of superconducting materials, e.g., MgB_2, Fe-based pnictides, heavy fermion compounds, etc. [1, 2]. It is considered that each band in a multi-band superconductor has a condensate with an amplitude and phase that weakly interacts with the condensates of the other bands. This circumstance creates possibilities for observation of principally new effects of macroscopic phase coherence related to interference phenomena between wave functions of electron superfluid condensates formed in different zones [2–4]. One of the striking phenomena of this type concerns formation of the fractional flux vortices in the mixed state of multiband superconductors [5]. It turns out that Abrikosov vortices in multiband superconductors at definite conditions can be decomposed into the fractional flux vortices formed by electron superfluid condensates of separate electron zones.

In the present work, we examine the possibility of such fractional flux vortices formation during the dynamical magnetization process when vortices enter inside superconductor overcoming the surface edge barrier in the applied external magnetic field. For the sake of simplicity, we explore the case of isotropic two-band superconductor with different characteristic parameters of superfluid condensates in two zones. The flux entry associated with vortex formation and proliferation inside the two-band superconductor mesoscopic cylinder settled in a parallel magnetic field is studied on the base of time-dependent Ginzburg–Landau (TDGL) theory. Solution of TDGL equations and subsequent modeling is performed numerically by use of the finite element method. We have explored the time evolution for the fractional vortices penetration process and their subsequent dynamics inside the specimen. Two types of geometry are considered for the cross-section of the specimen: (i) a circular disk and (ii) a circular disk with a triangular cutout. Obtained results indicate that magnetic flux penetrates inside the specimen in form of fractional vortices when they overcome the edge barrier, which may be different for these two vortex types. Inside the specimen, fractional vortices move in order to arrange; vortices of different types attract to each other and try to stick together thus forming composite vortices with the whole flux quantum value ϕ_0.

2. Fractional flux vortices in two-band superconductors

Existence of two electron zones with weakly interacting superfluid condensates of Cooper pairs allows to suggest that there are fractional flux vortices formed be superfluid electrons in each of these two zones. First it was suggested by Babaev [5] and later discussed in many other works (for a review see, e.g., [2, 4]). In this section, the main features of such fractional flux vortices are discussed on the base of the Ginzburg–Landau theory for isotropic two-band superconductor with a weak Josephson coupling of electron superfluid condensates in two zones [2–4]:

$$F = \sum_{j=1,2}\left[\alpha_j\left|\psi_j\right|^2 + \frac{\beta_j}{2}\left|\psi_j\right|^4 + \frac{1}{2m_j}\left|\left(-i\hbar\nabla - \frac{2e}{c}\mathbf{A}\right)\psi_j\right|^2\right]$$

$$+\frac{1}{8\pi}(\nabla\times\mathbf{A})^2 + \gamma\left(\psi_1\psi_2^* + \psi_2\psi_1^*\right), \quad (1)$$

$$J_s = \sum_{j=1,2}\left[-\frac{i\hbar e}{m_j}\left(\psi_j^*\nabla\psi_j - \psi_j\nabla\psi_j^*\right) - \frac{4e^2}{m_j c}\left|\psi_j\right|^2\mathbf{A}\right], \quad (2)$$

here F is the Ginzburg–Landau free energy functional, J_s is the supercurrent density, $\mathbf{A}$ is the vector-potential of magnetic field, m_j is electron mass, and ψ_j is the order parameter of superconducting state in the jth zone ($j = 1, 2$). The superconducting state in each of two zones may be characterized by specific values of the coherence length ξ_j and London penetration depth λ_j:

$$\xi_j = \sqrt{\frac{\hbar^2}{2m_j\left|\alpha_j\right|}},\ \lambda_j = \sqrt{\frac{m_j c^2}{16\pi e^2\psi_{j0}^2}},\ \lambda^{-2} = \sum_{j=1,2}\lambda_j^{-2}, \quad (3)$$

here λ is the total penetration depth for the two-band superconductor.

In what follows we will consider the London limit: $\xi_j << \lambda_i$ ($i, j = 1, 2$). For this case, one can assume:

$$\psi_j(r) = \psi_{j0} = \mathrm{const},\ \psi_{j0} = \sqrt{\frac{\left|\alpha_j\right|}{\beta_j}},\ \psi_j(r) = \psi_{j0}e^{i\theta_j(r)}, \quad (4)$$

and the supercurrent density (2) takes the form:

$$J_s = \sum_{j=1,2}\frac{2e\hbar\psi_{j0}^2}{m_j}\left(\nabla\theta_j - \frac{2\pi}{\phi_0}\mathbf{A}\right). \quad (5)$$

If the magnetic flux Φ is located within the restricted area inside the two-band superconductor, then integrating (5) along the closed trajectory surrounding this area at distances larger than λ (so that J_s goes to zero over this trajectory), one gets:

$$\Phi = \frac{\psi_{10}^2\frac{n_1}{m_1} + \psi_{20}^2\frac{n_2}{m_2}}{\psi_{10}^2\frac{1}{m_1} + \psi_{20}^2\frac{1}{m_2}}\phi_0, \quad (6)$$

here n_1 and n_2 are integers which arise due to integration of the $\nabla\theta_j$ term in (5) over the closed trajectory: $\oint\nabla\theta_j dl = 2\pi n_j$.

So, the flux Φ in (6) is quantized with flux quanta values ϕ_1 and ϕ_2:

$$\phi_1 = \phi_0\frac{\psi_{10}^2\frac{1}{m_1}}{\psi_{10}^2\frac{1}{m_1} + \psi_{20}^2\frac{1}{m_2}},\ \phi_2 = \phi_0\frac{\psi_{20}^2\frac{1}{m_2}}{\psi_{10}^2\frac{1}{m_1} + \psi_{20}^2\frac{1}{m_2}}. \quad (7)$$

These fractional flux quanta are associated with vortices, formed by electron superfluid condensates in two zones, and their values can be written in form:

$$\phi_1 = \phi_0\frac{\lambda_1^{-2}}{\lambda_1^{-2} + \lambda_2^{-2}} = \phi_0\frac{\lambda^2}{\lambda_1^2},\quad \phi_2 = \phi_0\frac{\lambda_2^{-2}}{\lambda_1^{-2} + \lambda_2^{-2}} = \phi_0\frac{\lambda^2}{\lambda_2^2},$$

$$\lambda^2 = \frac{\lambda_1^2\lambda_2^2}{\lambda_1^2 + \lambda_2^2}. \quad (8)$$

The free energy density functional for the two-band superconductor in the London limit can be written as:

$$F_L = \frac{1}{8\pi}\left[\sum_{j=1,2}\lambda_j^{-2}\left(\mathbf{A} - \frac{\phi_0}{2\pi}\nabla\theta_j\right)^2 + \left(\nabla\times\mathbf{A}\right)^2\right] + \bar{\gamma}\cos\left(\theta_1 - \theta_2\right). \quad (9)$$

The free energy of the whole sample is given by:

$$\Im = \int d^2r\, F_L(r).$$

Using the variation procedure $\delta\Im/\delta\mathbf{A} = 0$, one gets the London–Maxwell equation [4, 6]:

$$\lambda^2\nabla\times\mathbf{H} = -\mathbf{A} + \frac{\phi_1}{2\pi}\nabla\theta_1 + \frac{\phi_2}{2\pi}\nabla\theta_2. \quad (10)$$

Taking the *rotor* operation from both parts in (10) one obtains equation for the distribution of magnetic field, created by fractional vortices settled at positions $R_{1,k}$ and $R_{2,m}$ for both types of these vortices, respectively:

$$\lambda^2\nabla\times\nabla\times\mathbf{H} + \mathbf{H} = \mathbf{n}_z\left[\phi_1\sum_k\delta\left(\mathbf{r} - \mathbf{R}_{1,k}\right) + \phi_2\sum_m\delta\left(\mathbf{r} - \mathbf{R}_{2,m}\right)\right]. \quad (11)$$

We assume that the magnetic field, as well as the axis of vortices is oriented along the z axis, so that $\mathbf{R}_{1,k}$ and $\mathbf{R}_{2,m}$ denote vortex positions in the xy-plane.

Substitution of expression (10) for $\mathbf{A}$ in Eq. (9) allows to calculate the free energy density and energetic characteristics of fractional vortices, namely their self-energy and the interaction energy for vortices from the same and also from different electron zones [4, 6]. It turns out that fractional vortices from the same electron zone repel each other and their interaction energy is given by:

$$V_{\text{intra}}(r_{l,ij}) = \frac{\phi_l^2}{8\pi^2\lambda^2} K_0\left(\frac{r_{l,ij}}{\lambda}\right) - \frac{\phi_1\phi_2}{8\pi^2\lambda^2}\ln(r_{l,ij}), \quad \mathbf{r}_{l,ij} \equiv \mathbf{r}_{l,i} - \mathbf{r}_{l,j}. \tag{12}$$

Meanwhile, vortices formed by electron condensates from different zones attract each other and try to merge and form the composite vortex with a whole flux quantum ϕ_0. The interaction between fractional vortices, corresponding to this attraction is given by:

$$V_{\text{inter}}(r_{12,ij}) = \frac{\phi_1\phi_2}{8\pi^2\lambda^2}\left[K_0\left(\frac{r_{12,ij}}{\lambda}\right) + \ln(r_{12,ij})\right], \quad \mathbf{r}_{12,ij} \equiv \mathbf{r}_{1,i} - \mathbf{r}_{2,j}. \tag{13}$$

Thus, vortices in two-band superconductors preferably exist in form of composite vortices which are quite similar to those in single-band superconductors. Nevertheless, it's believed that at some conditions these composite vortices can be decomposed into fractional flux vortices. One of these possibilities is discussed in the next section.

3. Entrance of fractional flux vortices inside a two-band superconductor: TDGL modeling

Here we explore a possibility for the fractional vortices emergence inside a mesoscopic two-band superconductor cylinder in applied longitudinal magnetic field. The separation of a composite vortex, bearing the total flux quantum ϕ_0, into fractional flux vortices can proceed during the penetration of magnetic flux from outside in the interior of two-band superconductor due to different values of the Bean–Levingston surface barrier for fractional vortices formed by superfluid condensates from two electron zones [6]. We investigate the dynamics of entry and subsequent proliferation of fractional vortices inside the two-band superconductor by numerical solution and modeling of the time-dependent Ginzburg–Landau equations (TDGL) [7, 8]. For the two-band superconductor these equations can be written in the form [4, 9, 10]:

$$\frac{\hbar^2}{2m_j D_j}\left(\frac{\partial}{\partial t} + i\frac{2e}{\hbar}\varphi\right)\psi_j = -\frac{\delta F}{\delta\psi_j^*}, \ (j = 1, 2)$$

$$\sigma\left(\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \nabla\varphi\right) = -\frac{\delta F}{\delta \mathbf{A}}, \tag{14}$$

where F is the Gizburg–Landau free energy functional, given by Eq. (1), φ is electric field potential, D_j and m_j are electron diffusion coefficient and mass for jth zone; σ is the normal state conductivity at $T \approx T_c$.

We use a dimensionless form of the system (14) with the zero electric potential gauge:

$$\frac{\partial\psi_1}{\partial t} + \left(|\psi_1|^2 - 1\right)\psi_1 + \left(\frac{i}{\kappa_1}\nabla + \mathbf{A}\right)^2 \psi_1 + \gamma\psi_2 = 0,$$

$$\frac{\partial\psi_2}{\partial t} + \left(|\psi_2|^2 - 1\right)\psi_2 + \frac{\xi_2^2}{\xi_1^2}\left(\frac{i}{\kappa_1}\nabla + \mathbf{A}\right)^2 \psi_2 + \gamma\nu^2\psi_1 = 0,$$

$$\sigma\frac{\partial \mathbf{A}}{\partial t} = \nabla\times\mathbf{H}_e - \nabla\times\nabla\times\mathbf{A} + \frac{i}{2\kappa_1}\left(\psi_1\nabla\psi_1^* - \psi_1^*\nabla\psi_1\right)$$

$$+\frac{i}{2\nu\kappa_2}\left(\psi_2\nabla\psi_2^* - \psi_2^*\nabla\psi_2\right) - \left(|\psi_1|^2 + \frac{\lambda_1^2}{\lambda_2^2}|\psi_2|^2\right)\mathbf{A} \tag{15}$$

Here, dimensionless length is measured in units of λ_1, time is measured in units of $\frac{\xi_1^2}{D_1}$; $\nu = \frac{\lambda_2\xi_2}{\lambda_1\xi_1}$.

For numerical solution equations (15) should be completed by boundary and initial conditions, which are as follows:

$$\nabla\psi_j \cdot \mathbf{n}_s\big|_S = 0, \quad \mathbf{A}\cdot\mathbf{n}_s\big|_S = 0, \quad \nabla\times\mathbf{A}\times\mathbf{n}_s\big|_S = \mathbf{H}_e\times\mathbf{n}_s\big|_S$$

$$\psi_j(\mathbf{r}, t\to 0) = \psi_{j0}(\mathbf{r}), \qquad \mathbf{A}(\mathbf{r}, t\to 0) = \mathbf{A}_0(\mathbf{r}).$$

For carrying out the numerical solution and modeling of TDGL equations we have chosen the following parameters, characterizing the electron superfluid condensates in two zones, namely:

The Ginzburg–Landau parameters $\kappa_1 = \lambda_1/\xi_1 = 10$, $\kappa_2 = \lambda_2/\xi_2 = 2.5$, also $\lambda_1/\lambda_2 = 2$, $\xi_1/\xi_2 = 0.5$, $\sigma = 1$, $\psi_1(\mathrm{r}, t\to 0) = 0.8 + 0.6i$, $\psi_2(\mathrm{r}, t\to 0) = -0.8 - 0.6i$, $\gamma = 0.001$. For these superconducting state parameters the time evolution of magnetic flux distribution inside the two-band superconductor after the switching on the magnetic field was modeled for different applied external field values. In Figs. 1 and 2 these flux distributions are demonstrated for different external magnetic field values for some chosen dimensionless time moment ($t = 100$), while in Figs. 3 and 4 the time evolution of the magnetic flux related to the fractional vortices penetration inside the two-band superconductor is illustrated for some chosen values of applied magnetic field ($H = 1.9$ and $H = 2.2$, respectively). From the results presented in these figures it distinctly follows that magnetic flux enters inside the two-band superconductor in form of fractional flux vortices. Fractional vortices of the same zone repel from each other, while vortices from different electron zones attract to each other and have a tendency to merge and form the composite vortex bearing the whole magnetic flux quanta ϕ_0. An additional

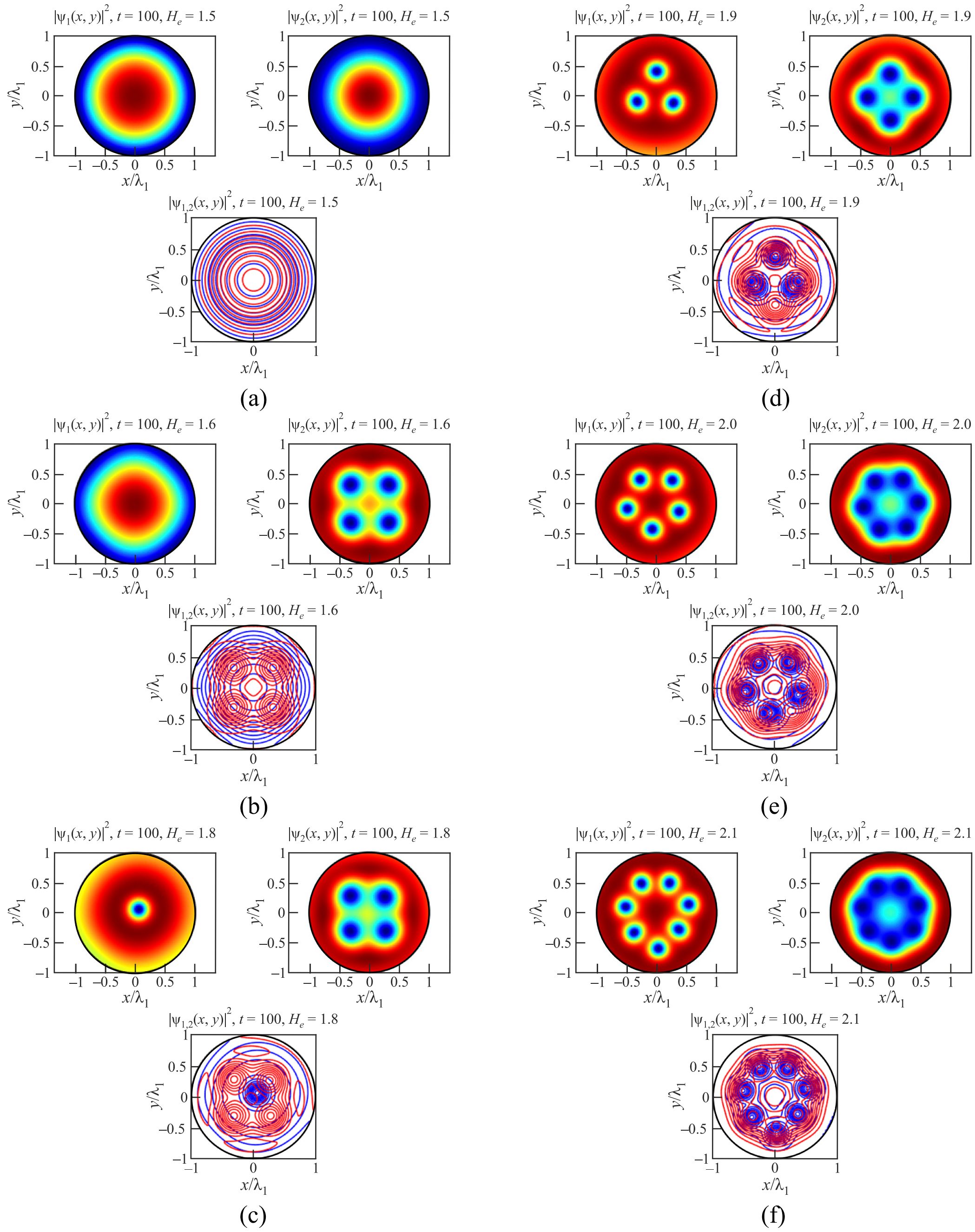

Fig. 1. Penetration of fractional vortices inside the mesoscopic superconducting cylinder at different external magnetic field values, *H*: 1.5 (a), 1.6 (b), 1.8 (c), 1.9 (d), 2.0 (e), 2.1 (f) accordingly to the numerical modeling of TDGL by use of the finite element method. Vortex patterns are calculated at the definite time moment ($t = 100$ in dimensionless units) after the magnetic field is turned on and vortices started their entry inside the specimen with an overcome of the Bean–Levingston barrier.

feature of magnetic flux penetration inside a superconductor is demonstrated in Figs. 2 and 4. One can see that vortices formed in superfluid condensates from both zones penetrate inside the superconductor through the sharp corner of the triangle cutout. The latter occurs because of the Meissner current concentration near the triangle apex. This current creates the Lorentz force which pushes vortices in the interior of superconductor.

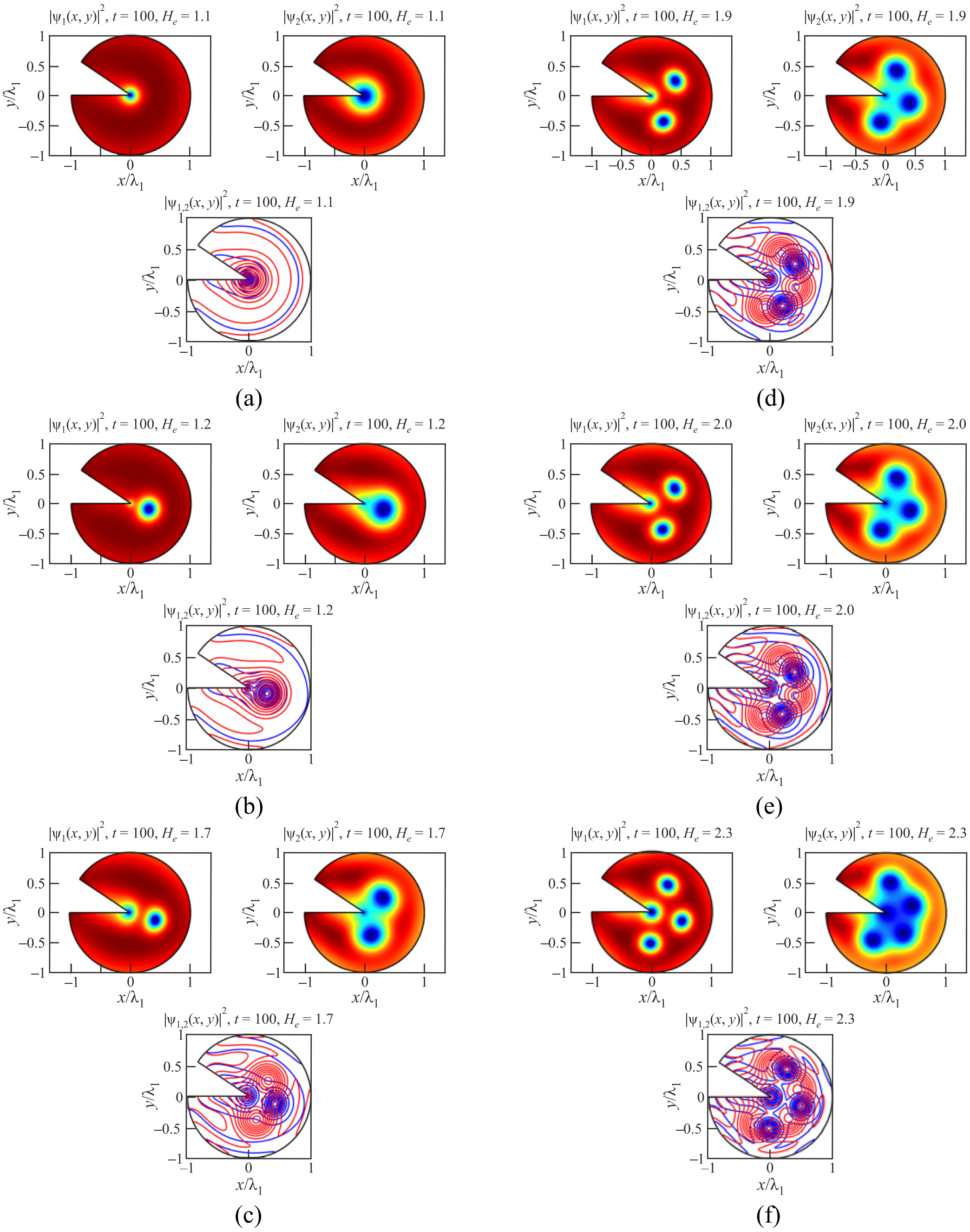

Fig. 2. Penetration of fractional vortices inside the mesoscopic superconducting cylinder with a triangular cutout at different external magnetic field values, H: 1.1 (a), 1.2 (b), 1.7 (c), 1.9 (d), 2.0 (e), 2.3 (f) accordingly to the numerical modeling of TDGL by use of the finite element method. Vortex patterns are calculated at the definite time moment ($t = 100$ in dimensionless units) after the magnetic field is turned on. Vortices entry inside the specimen preferably through the apex of the triangular cutout where the current stream concentrates.

4. Conclusion

Numerical modeling of magnetic flux penetration and further proliferation in the interior of two-band superconductor was performed in the present work by the solution of TDGL equations using the COMSOL Multiphysics program. This study has revealed that magnetic flux penetrates inside the two-band superconductor in form of fractional flux quanta, related to vortices formed by electron condensates of two zones. From previous theoretical studies it was known that fractional vortices are thermodynamically unstable and can exist only in mesoscopic samples or near the specimen surface [6, 11, 12]. Moreover, fractional flux vortices from different zones attract to each other [see Eq. (13)] and try to merge and form the composite vortex with a whole flux quantum ϕ_0. In [13] it was suggested that composite vortices in two-band superconductors in the flux flow state can dissociate and form fractional vortices due to the disparity of the vortex viscosity and flux for fractional vortices from different bands. In the present work, we

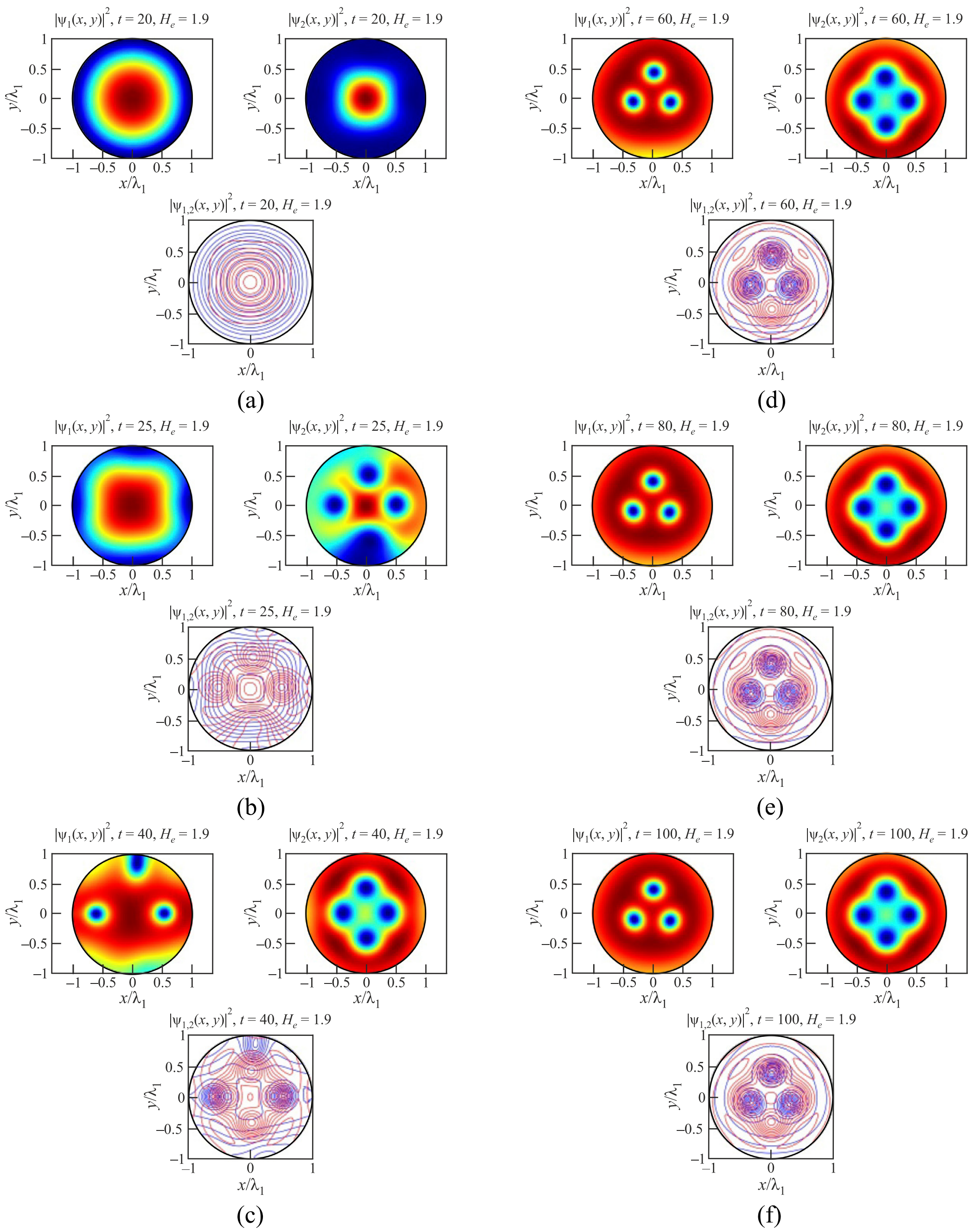

Fig. 3. Time evolution of magnetic flux penetration inside the circular two-band superconductor settled in external magnetic field ($H_e = 1.9$) at different time moments after the turning on the magnetic field, t: 20 (a), 25 (b), 40 (c), 60 (d), 80 (e), 100 (f).

argue that fractional vortices can arise and exist during their entrance and subsequent proliferation inside two-band superconductor, at least during some time of the magnetization process. In this case separation of the composite vortices with a whole flux quantum ϕ_0 into fractional ones can arise due to the different thermodynamic critical field $H_{c,j}$ values for condensates from two zones and, correspondingly, due to the different threshold field values for vortex penetration through the edge barrier [6]. The other reason for a such kind separation may be related to the different flux and viscosity values for fractional vortices affecting their proliferation inside the specimen during the magnetization dynamic process. This is the case considered in the present work.

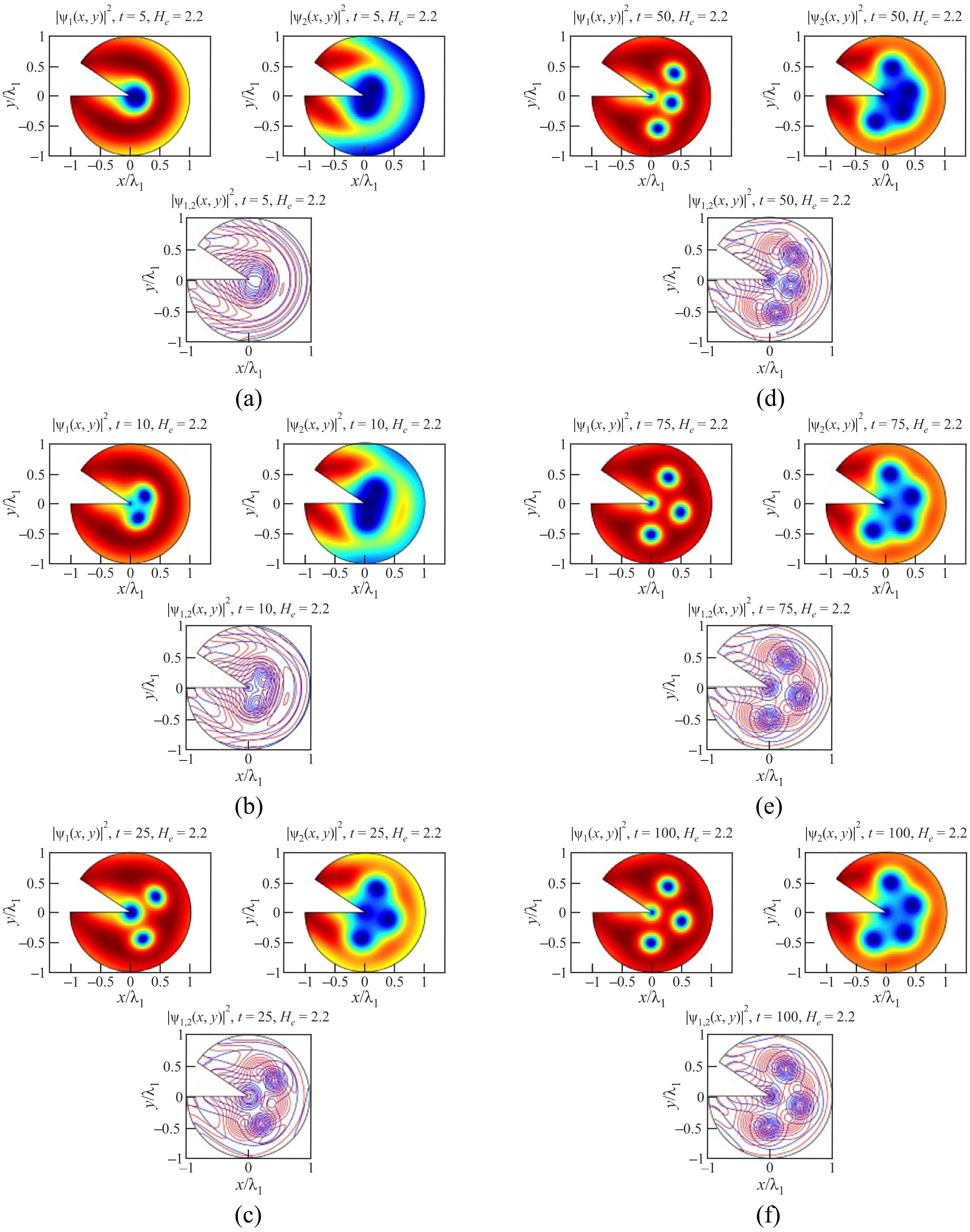

Fig. 4. Time evolution of magnetic flux penetration inside the two-band superconductor specimen with a cross-section in form of the circle with a triangular cutout, settled in external magnetic field ($H_e = 2.2$) at different time moments after the turning on the magnetic field, t: 5 (a), 10 (b), 25 (c), 50 (d), 75 (e), 100 (f).

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Динаміка фракційних вихорів у двозонних надпровідниках

A. M. Kutsyk, A. L. Kasatkin, A. A. Kordyuk

Чисельним розв'язком залежних від часу рівнянь Гінзбурга–Ландау (TDGL) досліджено входження фракційних вихорів та їх подальша динаміка всередині двозонного надпровідника. Розглянуто випадок, коли надплинні електронні конденсати двох зон характеризуються досить різними параметрами, такими як довжина когерентності ξ_i та лондонівська глибина проникнення λ_i, що призводить до різних критичних магнітних полів $H_{c,i}$ та фракційних значень квантів магнітного потоку, який пов'язаний із вихорами у надпровідному стані в цих двох зонах. Чисельні розв'язки рівнянь TDGL при різних значеннях зовнішнього магнітного поля з подальшим математичним моделюванням проникнення магнітного потоку виконано для цього випадку методом скінченних елементів. Досліджено часову еволюцію процесу проникнення фракційних вихорів та їх подальшу динаміку всередині зразків для двох геометрій: кругового диска та кругового диска з трикутним вирізом. Отримані результати свідчать про те, що магнітний потік проникає всередину зразка у вигляді фракційних вихорів, коли вони можуть подолати крайовий бар'єр, який, в загальному випадку, є різним для цих двох типів вихорів. Тому при збільшенні зовнішнього магнітного поля у двозонний надпровідник спочатку проникають вихори, для яких висота бар'єру є меншою (тобто, вихори із зони з меншим критичним полем $H_{c,i}$), тоді як фракційні вихори іншої зони починають своє проникнення при більш високій величині зовнішнього поля. Інший механізм утворення дробових вихорів під час їх входу в двозонний надпровідник пов'язаний з різницею в значеннях фракційних квантів потоку та коефіцієнтів в'язкості, які визначають швидкість поширення вихорів всередині зразку. Всередині зразку фракційні вихори рухаються, намагаючись упорядкуватися. Вихори з різних електронних зон притягуються один до одного, злипаються, утворюючи композитні вихори з цілим значенням квантів всього потоку $\phi_0 = h/2e$.

Ключові слова: двозонний надпровідник; вихори фракційного потоку, довжина когерентності, глибина проникнення, критичне поле, крайовий бар'єр.