

## Dynamics of a permanent magnet levitating above a high- $T_c$ superconductor

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Received 31 July 1990

The oscillation modes of a small spherical permanent magnet (1.7 mm diameter) above a flat high- $T_c$  superconductor are investigated. From the attenuation and nonlinearity of the vertical oscillation we obtain the viscosity of the vortex lattice and the onset of hysteretic damping, which indicates depinning of flux lines. The shape of the potential well in which the magnet floats, rotates, and oscillates is found to depend on the magnetic history of the superconductor.

In view of the new physical phenomenon – the free suspension of a permanent magnet (PM) in a finite interval of stability above and below a high-temperature superconductor (HTSC) [1–3], the investigation of the macroscopic dynamics of this levitation is now being carried out [3–5]. In this paper we investigate the free oscillations of a small PM above a HTSC, in particular, the resonance frequency  $\nu_0$  and the damping rate  $\gamma = \Delta\nu$  as a function of the amplitude of the driving force  $f$ . We define  $\Delta\nu$  as the width of the resonance curve at the height  $A_{\max}/\sqrt{2}$  where  $A_{\max}$  is the maximum amplitude of the forced oscillations of the PM when the frequency is scanned at a given amplitude of the driving force  $f$ . Various technical characteristics of HTSCs are easily obtained in this way, namely the viscosity  $\eta$  of the vortex motion, the pinning force, and the energy loss  $W_1$  in the system in an alternating magnetic field. These characteristics are mainly determined by *irreversible* processes during the motion of vortex filaments of magnetic flux in the superconductor, namely, either by the “stripping” of vortices from the pinning centers (this “depinning” occurs at large amplitudes  $\Delta B_a$  for the applied AC field and is characterized by the area of a hysteresis loop) or, at lower amplitudes, by the viscous flow of the vortex structure (flux flow) or by thermally activated flux motion (flux creep and thermally assisted flux flow [6–8]). Low-amplitude processes occur in the ‘quasi-

elastic” sections of the hysteretic magnetization curve, where  $\Delta M \approx -\Delta B_a$  since the magnetic flux is nearly ideally pinned and thus  $B \approx \text{constant}$ . (If demagnetization effects may be disregarded one has  $M = B - B_a$ ).

This work proposes a method for studying the characteristics of HTSC-materials in an alternating magnetic field in both the hysteresis and “quasi-elastic” areas. The investigations were carried out at a temperature of 80 K. Our PM of  $\text{SmCo}_5$  had the shape of a spheroid with excentricity  $\approx 0.6$  and with mass  $m \approx 0.021$  g and magnetic moment  $\mu \approx 1.2$  G  $\text{cm}^3$ . It levitated above the HTSC at a height of  $x_0 \approx 0.3$  cm with  $\mu$  orientated parallel to the HTSC surface. Our HTSC had the shape of discs with 2 cm diameter and 0.3 cm thickness made from the cast uniphase  $\text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_{6.95}$  with  $T_c \approx 93$  K and  $\Delta T_c \approx 2$  K. From X-ray data this  $\text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_{6.95}$  has a perfect orthorhombic structure with the parameters  $a_0 = 3.840 \pm 0.004$  Å,  $b_0 = 3.860 \pm 0.004$  Å,  $c_0 = 11.669 \pm 0.004$  Å and an orthorhombicity ratio  $b_0/a_0 \approx 1.005$ . The oxygen deficiency  $\delta$  in  $\text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_{7-\delta}$  was determined by a mass-spectrometer with oxygen-calibrated volume [9].

Figure 1 shows the scheme of the experiment. The forced oscillations of the PM with frequency  $\nu = \omega/2\pi$  were excited by the AC coil C located above the magnet coaxially with the  $x$ -axis. These oscillations are a sensitive detector of the vortex lattice in the

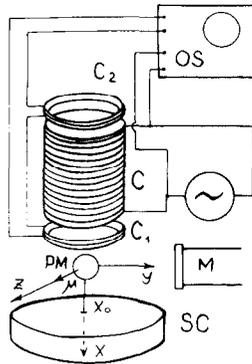


Fig. 1. Experimental scheme for the investigation of the free oscillation of a permanent magnet (PM) freely levitating above an  $Y_1Ba_2Cu_3O_{6.95}$  high- $T_c$  superconductor (HTSC).

HTSC. The oscillating PM is the source of an alternating magnetic field which is registered by two coils  $C_1$ ,  $C_2$  with an identical number of turns connected in opposition and positioned at the ends of the coil  $C$  coaxially with the  $x$ -axis. A double-beam oscillograph (OS) shows simultaneously the free oscillation of the magnet and the phase difference  $\Delta\varphi$  between the driving field and this oscillation. The resonance point  $\nu = \nu_0$  is defined by the phase shift  $\Delta\varphi = \pi/2$ . In our experiment three oscillations modes with different resonance frequencies were observed: (a) translation along the  $y$ -axis at  $\nu_0^y \approx 17$  Hz; (b) translation along the  $x$ -axis at  $\nu_0^x \approx 25$  Hz; (c) rotation about the  $y$ -axis at a frequency of  $\approx 70$  Hz. These resonance frequencies belong to low amplitudes of these in general *nonlinear* oscillations. The analysis of the amplitude–frequency relationship  $A(\nu)$ , for convenience of experimental observation and simplicity of the further considerations, was carried out in the vicinity of the frequencies  $\nu_0^x$  of the oscillations along the vertical  $x$ -axis over an amplitude range  $A \approx 10^{-3}$  cm to  $A \approx 10^{-1}$  cm. At large amplitudes the registration of the PM oscillations could be carried out both electronically by using an oscilloscope (OS) and optically by a microscope (M). The microscope allowed to calibrate the oscilloscope. Small amplitudes were only measured electronically.

Figure 2 shows the amplitude  $A(\nu^x)$  of the PM levitating at the height  $x_0 \approx 0.3$  cm above a disk of  $Y_1Ba_2Cu_3O_{6.95}$  for various values of the external force

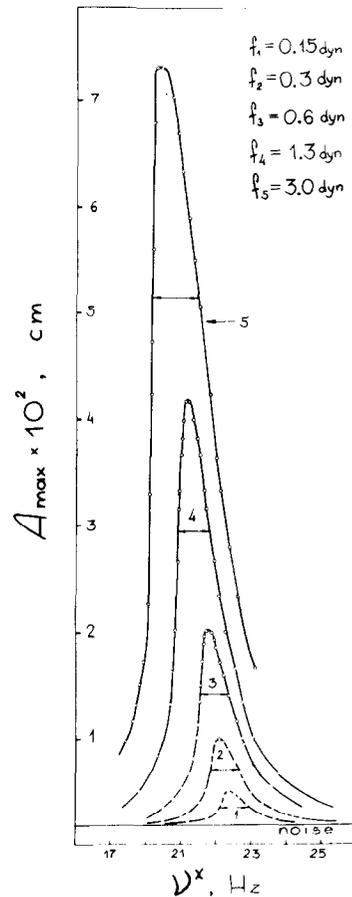


Fig. 2. Several amplitude-frequency characteristics for the translational mode  $\nu^x$  of the magnet above a disk of  $Y_1Ba_2Cu_3O_{6.95}$  at various values of the driving force  $f$ . The amplitude is obtained from the current in the coil  $C$ , fig. 1.

$f$  determined by the current in the coil  $C$ . In this case (see area A in fig. 3(I)) the damping factor  $\gamma$ , defined as the width of the resonance curve (fig. 2) at the height  $A_{\max}/\sqrt{2}$ , is practically constant. The motion of the vortex lattice in this region can be described as motion in a viscous medium by the formula  $F_v = -\eta v V_0$ , where  $F_v$  is the viscous frictional force exerted on a volume  $V_0$  of the vortex lattice moving with velocity  $v$  and  $\eta$  the (volume) viscosity coefficient, which may be estimated as follows. According to the definition the quality factor of the oscillating system can be written as

$$Q = (\nu_0^x / \gamma) = 2\pi(W / W_1), \quad (1)$$

where  $W = \frac{1}{2} mA_{\max}^2 (2\pi\nu_0^x)^2$  is the energy stored in the PM-HTSC system and  $W_1 = \oint F_v dx$  is the energy loss during one period. Averaging over all vortices we obtain  $W_1 = \eta \langle v^2 \rangle V_0 / \nu = 2\pi^2 \eta \nu_0^x a^2 V_0$ , where  $a^2$  is the mean square oscillation amplitude of the vortex lattice and  $V_0$  is the volume of the superconductor in which the main energy dissipation occurs. Due to the steeply decreasing dipolar magnetic field  $H \sim 1/x^3$  the volume  $V_0 \approx x_0^3$ , and the corresponding surface area on the HTSC  $S_0 \approx x_0^2$  [cf. eq. (4) below]. Substituting  $W$  and  $W_1$  in eq. (1) we obtain

$$\eta = \frac{2\pi m \gamma}{V_0 \alpha^2}, \quad (2)$$

where  $\alpha = a/A_{\max}$ . Estimating the viscosity coefficient in the area A of fig. 3 by using (2) and the experimental data from this work,  $\nu \approx 1.6$  Hz,  $B_a \approx 15$  G,  $V_0 \approx x_0^3 \approx 2.7 \times 10^{-2}$  cm<sup>3</sup>,  $x_0 \approx 0.3$  cm,  $m = 0.021$  g we obtain  $\eta \approx (800/\alpha^2)$  kg s<sup>-1</sup> m<sup>-3</sup>. A quantitative estimation of  $\eta$  is difficult since  $a$  is not well known. Note that in this experiment within stable equilibrium of the PM,  $a$  cannot be larger than  $a_0$ , the main distance between the vortices, otherwise the vortices are depinned and the damping is no longer of the viscous type but becomes amplitude dependent (hysteretic damping) [10]. At  $B = 15$  G one has  $a_0 = 3.9$   $\mu$ m whereas  $A_{\max}$  10 to 30  $\mu$ m, hence  $\alpha \ll 1$ . Thus, we can conclude that in our experiment we had  $\eta \gg 10^4$  kg s<sup>-1</sup> m<sup>-3</sup>.

The volume viscosity  $\eta$  is related to the flux flow

resistivity  $\rho_{FF} = B^2/\eta$  [8]. For our experiment we get  $\rho_{FF} = (2.8 \alpha^2) \times 10^{-7}$   $\Omega$ cm. This very small resistivity is consistent with the simple relationship  $\rho_{FF} \approx \rho B/B_{c2}(T) \ll \rho_n$  where  $\rho_n \approx 10^{-6}$   $\Omega$ cm is the normal resistivity of the HTSC at the same temperature and  $B_{c2} > 10^6$  G is the upper critical field [8].

The magnified section of the curve ( $\gamma$  versus  $A_{\max}$ ) (fig. 3 (II)) shows that within the area A there occurs a small (about 6%) but sharp change of  $\gamma$  at frequencies  $\nu_0^x$  near 24.5 Hz and 23 Hz. As in this case there are no peculiarities of the  $\nu_0^x$  versus  $A_{\max}$  dependence, variations in  $\gamma$  apparently do not influence the elastic properties of the vortex-pin system but the depinning process changes slightly at these amplitudes. This conclusion is supported by the fact that such peaks in  $\gamma$  are not observed in the amplitude interval A even when this is passed repeatedly.

Taking into account that in the area A in fig. 3-I one has  $\Delta B_a \sim 0.1$  G  $< B_{c1}$ , ( $B_{c1}$  is the lower critical field) and that after switching off the alternating force  $f$  the PM returns into its initial equilibrium state, we may assume that in the area A weak flux creep occurs in a quasi-elastic area. This thermally activated depinning from weak pinning centres results in a decrease of the averaging pinning force and explains the anharmonicity observed in this case in the system PM-HTSC within the area A of fig. 3(I), where  $\gamma$  is constant. The role of thermal depinning is experimentally supported by additional data obtained with powder-compacted samples of Y<sub>1</sub>Ba<sub>2</sub>Cu<sub>3</sub>O<sub>7.0</sub>

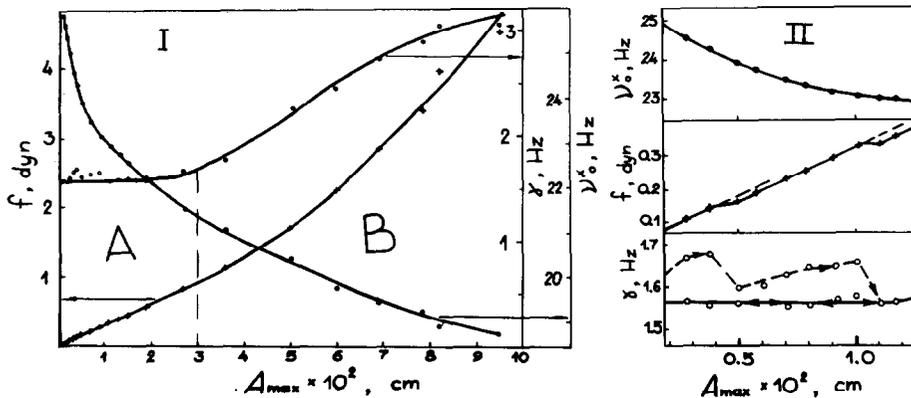


Fig. 3. (I) The driving force  $f$ , the resonance frequency  $\nu_0^x$ , and the damping factor  $\gamma = \Delta\nu^x$  of the translational mode along the  $x$ -axis, fig. 1, plotted vs. the resonance amplitude of the oscillations  $A_{\max} = A(\nu_0^x)$ .  $\Delta\nu^x$  is the width of the resonance curve (fig. 2) at  $1/\sqrt{2}$  times the maximum amplitude  $A_{\max} = A(\nu_0^x)$ . (II) The same  $\gamma = \Delta\nu^x$ ,  $f$  and  $\nu_0^x$  vs.  $A_{\max} = A(\nu_0^x)$  on an enlarged scale.

(the size of grains in the powder is 3 to 10  $\mu\text{m}$ ), where compared to the cast ones, the relative portion of weak pinning centres is decreased at the expense of an increase of strong surface pinning. Here, under analogous experimental conditions an anharmonicity in the system PM-HTSC was not observed up to amplitudes of  $A_{\text{max}} \sim 10^{-2}$  cm.

With the increase of the amplitude of the driving force  $f$  in the area B in fig. 3(I), the damping  $\gamma$  begins to increase and the PM oscillation becomes *anharmonic*. In this case a decrease of  $\nu_0^*$  with increasing amplitude  $A_{\text{max}}$  is caused both by stripping off some vortices from the pinning centres, which results in a softening of the vortex-pin system (similar as in the area A in fig. 3(I)) and by the increase of  $\gamma$  according to the basic relationship  $\nu_0 = \sqrt{\nu_R^2 - \gamma^2/2}$  ( $\nu_R$  is the resonance frequency of frictionless oscillations [11]). An increase of  $A_{\text{max}}$  enhances the depinning. This conclusion is supported by the fact that an increase of  $A_{\text{max}}$  above the values given in fig. 2 (curve 5), where the amplitude of changing magnetic field is  $\Delta B_a \approx 20 \text{ G} > B_a$ , results in complete depinning of the vortex lattice and the PM occupies a new equilibrium state with height  $x'_0 > x_0$  ("floating up" of the magnet). The hysteresis found in the system PM-HTSC indicates that the shape of the potential well, unlike with the usual nonlinear oscillator, depends on the magnetic history.

In fig. 4 the energy  $W$  stored in the damped oscillation is plotted as a function of  $f^2$  where the force amplitude  $f = mA_{\text{max}}\omega_0 \cdot (2\pi\gamma)$  [11] is determined

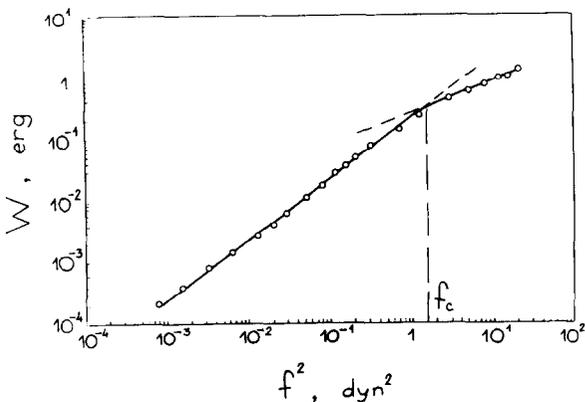


Fig. 4. The stored energy  $W$  of the oscillations versus the squared exciting force  $f^2$ .

from the experimental data  $A_{\text{max}}$ ,  $\omega_0 = 2\pi\nu_0$ , and  $\gamma$  (fig. 2). Clearly, two types of behavior can be distinguished:

$$W = \begin{cases} k_1 f^2, & f < f_c \quad (\text{viscous damping}) \\ k_2 f & f > f_c \quad (\text{hysteretic damping}) \end{cases} \quad (3)$$

corresponding to the areas A and B in fig. 3(I) ( $k_1 \approx 0.4 \text{ s}^2/\text{g}$ ,  $k_2 \approx 0.45 \text{ cm}$ ). Taking into account that  $W = f^2/m\gamma^2$  [11], we may conclude that in the area A one has  $k_1 = 1/m\gamma^2 = 0.4 \text{ s}^2/\text{g}$  and  $\gamma = \text{const}$ . In the area B,  $k_2 = f/m\gamma^2 = 0.45 \text{ cm}$ , and it should be  $\gamma \sim \sqrt{f}$ . From the relationship for the loss per unit area (cf.  $W_1$  from eq. (1) with  $W$  inserted),

$$\frac{W_1}{S_0} = \frac{4\pi^3 m \nu_0 \gamma A_{\text{max}}^2}{S_0}, \quad (4)$$

with  $S_0 \approx x_0^2$ , we can estimate the losses  $W_1/S_0$  in both the "quasi-elastic" (A) and hysteresis (B) regions. Under the experimental conditions  $\nu_0^* \sim 20 \text{ Hz}$  the losses caused by air resistance may be neglected since they are of only a few percent. We obtain for the quasi-elastic or viscous losses (area A)  $W_1/S_0 \approx 5.3 \times 10^{-11} \text{ J/cm}^2$  and for the hysteretic losses (area B)  $W_1/S_0 \approx 1.34 \times 10^{-6} \text{ J/cm}^2$ . The above considerations show that the investigation of the macroscopic dynamics of a magnet levitating above a type-II superconductor gives valuable insight into pinning and depinning of the flux lines inside the superconductor.

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