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Evolution of Bounded Majorana Pairs in Superconducting Net of Quantum Nanowires in $La_{0.15}Sm_{0.85}MnO_{3+}$ and $SmMnO_{3+}$ with Increase of External Magnetic Field; Relativistic Quantum Hall Effect

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The temperature dependence of the 'supermagnetization' $M(T)$ in the first Landau zone of $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ and in the magnetic field of 100 Oe has the shape of a Dirac's cone-like truncated hill with a flat top near $T \cong 4.6$ K. In external magnetic field with $H = 350$ Oe, a distinct magnetic response appears within the first Landau band with the shape of two spiky peaks near $T_{MZM} \cong 4.6$ K. These spiky features in the magnetic response arise from excited states containing either both only static magnetic fluxes and no mobile fermions, or from excited states, in which fermions are closely coupled to fluxes. An alternate permutation of the spiky double peaks and Dirac cones features of the magnetization $M(T)$ may be explained by the availability in this material of two hidden states of the chiral spin liquid. A further increase in the external magnetic-field strength to the value $H = 1$ kOe leads to the formation in the Landau zone with $n = 1$ of intensive truncated hill feature with a flat top near the average temperature $T \cong 4.6$ K. In external magnetic field with $H = 3.5$ kOe, only the step-like quantum oscillations of temperature dependences of 'supermagnetization' of incompressible quantum spinon liquid are found. Similar alternate permutations of $M(T)$ features are found in $SmMnO_{3+\delta}$, but in a different sequence. It is assumed that Majorana bound pairs are captured at the ends of quantum nanowires and form coherent ground states, which are very stable as to external influences. They are interesting tools for the construction of quantum computers due to their exotic non-Abelian exchange statistics.

Температурна залежність «супернамагнетованости» $M(T)$ у першій зоні Ландау $La_{0,15}Sm_{0,85}MnO_{3+\delta}$ у магнетному полі у 100 Е має форму усіченого пагорба, подібного до Діракового конуса, з пласкою вершиною поблизу $T \cong 4{,}6$ К. У зовнішньому магнетному полі з $H = 350$ Е в першій смузі Ландау з'являється чітка магнетна реакція у формі двох гострих піків поблизу $T_{MZM} \cong 4{,}6$ К. Ці гострі особливості в магнетній реакції виникають через збуджені стани, що містять або лише статичні магнетні потоки і не мають рухомих ферміонів, або через збуджені стани, в яких ферміони тісно пов'язані з потоками. Чергове переустановлення гострих подвійних піків і особливостей Діракових конусів намагнетованости $M(T)$ можна пояснити наявністю у цьому матеріялі двох прихованих станів хіральної спінової рідини. Подальше збільшення напружености зовнішнього магнетного поля до значення $H \approx 1$ кЕ привело до утворення в зоні Ландау з $n \approx 1$ інтенсивної усіченої пагорбоподібної особливости з пласкою вершиною поблизу середньої температури $T = 4{,}6$ К. У зовнішньому магнетному полі з $H \approx 3{,}5$ кЕ було виявлено лише ступінчасті квантові коливання температурних залежностей «супермагнетування» нестисливої квантової спінонної рідини. Подібні альтернативні перестановки $M(T)$-особливостей було виявлено в $SmMnO_{3+\delta}$, але в іншій послідовності. Передбачається, що зв'язані Майоранові пари захоплюються на кінцях квантових нанодротів і утворюють когерентні основні стани, які дуже стійкі щодо зовнішніх впливів. Вони є цікавими інструментами для побудови квантових комп'ютерів завдяки своїй екзотичній неабелевій обмінній статистиці.

Key words: Majorana zero modes, massless 2*D* Dirac's fermions, Dirac's cone-like truncated hill of $M(T)$, two spiky peaks of $M(T)$, step-like quantum oscillations of $M(T)$, Landau quantization of massless Dirac's fermions, net of quantum nanowires.

Ключові слова: Майоранові нульові моди, безмасові двовимірні Діракові ферміони, Діраків конусоподібний усічений пагорб $M(T)$, два гострі піки $M(T)$, ступінчасті квантові коливання $M(T)$, квантування Ландау безмасових Діракових ферміонів, сітка квантових нанодротів.

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1. INTRODUCTION

It is well known that, in quantum physics, the Schrödinger equation is used to describe the wave function of a quantum-mechanical system. It was proposed in 1926 by Schrödinger [1] and is conceptually the quantum counterpart of Newton's second law describing the path of a physical system over time depending of the initial conditions. Dirac proposed a new relativistic wave equation in 1930 [2], where the wave function ψ has four components. A direct consequence of this formulation is the concept of 'particle and antiparticle' also proposed by Dirac. When considering spin S particles (fermion), the positive-energy solutions of Dirac equation describe electrons, and negative-energy solutions describe particles with the same mass and spin but opposite charge known as positrons. Their existence was later confirmed in 1932 by Anderson. Dirac's fermions are thus described by complex wave functions; the wave function of the positron (the antiparticle) is the complex conjugate of the wave function of the electron (the particle). In 1937, Ettore Majorana proposed a new solution for Dirac's equation excluding imaginary numbers and, thus, theoretically predicted a new class of fermions: the Majorana fermions (MFs) [3]. They correspond to the real wave-functions' solutions of Dirac's equation, and are thus particles, which are their own antiparticles. So far, there has been no evidence of the existence of MFs as elementary particles. In a theoretical paper from 2001 [4], Kitaev proposed to engineer Majorana operators in solid-state systems. They can be described as quasiparticles or mathematic operators, which have the same properties as Majorana fermions, and correspond to 'vortex' in condensed-matter systems. They are interesting tools for building quantum computers due to their exotic non-Abelian exchange statistics [5].

Indeed, all known bosons and fermions obey the following principle: when two indistinguishable particles exchange their positions, the ground state of the system remains unchanged. On the other hand, when two Majorana quasi-particles exchange positions, the system transition from one quantum ground state to another distinct one. This unique behaviour can be used to store and process information in pairs of Majorana quasi-particles spatially separated by a superconducting gap. Qubits would thus be topologically protected against local perturbations, which would increase their coherence time. Finally, two theoretician groups [6, 7] proposed in 2010 a simple recipe to engineer the Majorana pairs in nanowire devices: if a one-dimensional semiconductor nanowire with strong Rashba spin–orbit interactions is weakly coupled to a superconductor, Majorana quasi-particles should emerge at the ends of this hybrid system as a single state in the middle of the proximity-induced superconducting gap. Majorana quasi-particles have topological protection, making them more robust against errors and noise than conventional qubits. How does this relate to quantum computers? Researchers use Majorana fermions to create topological qubits. Information in these qubits is encoded not in individual particles, but in their collective states. Microsoft developed the Majorana 1 chip—the first device built using this new topological architecture. The goal of this approach is to create quantum computers, which are significantly more stable and less prone to errors than existing systems. In the future, such chips are expected to accommodate millions of qubits, enabling the solution of large-scale industrial problems.

According to Ref. [8], a Majorana fermion (MF) is a particle that is its own antiparticle. In the language of second quantization, this means that $\gamma = \gamma^\dagger$, *i.e.*, the fermionic operator γ squares to 1. The creation and annihilation operators can be written as a superposition of two Majorana operators, $a^\dagger = 1/\sqrt{2}(\gamma_1 + i\gamma_2)$, $a = \sqrt{2}(\gamma_1 - i\gamma_2)$. As such, they also fulfil the commutation identity $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$. The task at hand is to physically separate the two Majorana modes, γ_{2j} and γ_{2j-1}, that make up a single fermionic mode, such that phase errors corresponding to

$a_j^{\dagger}a_j=(1+i\gamma_{2j-1}\gamma_{2j})/2$ are unlikely to occur. If you put together, these properties would make the Majorana qubit immune to decoherence. These Majorana fermions can arise as quasi-particles in superconducting systems, which have been investigated in Ref. [8] in a one-dimensional chain first proposed by Kitaev [4]. Shown that they are bound to zero energy, making them Majorana zero modes (MZMs)—a more apt name given that they no longer obey fermionic statistics—where $[H, \gamma_i]=0$, with H being the Hamiltonian of the system (more realistically, this condition is 'relaxed' to $[H, \gamma_i]\approx e^{-x/\xi}$ [9], where x is the distance between the Majorana zero modes (MZMs) and ξ is the correlation length of the Hamiltonian. They obey non-Abelian statistics that enable the implementation of braid operations. This solves the final piece of the puzzle, where qubit operations are now intrinsically fault-tolerant due to their topological properties.

The so-called Majorana bound states arising on point defects have attracted great interest [10–19]. They can be interpreted as own antiparticles in the sense that, in the language of second quantization, the operators of creation and annihilation of bound states are equal to each other. This means that Majorana bound states carry both zero spin and zero charge. Majorana bound states arise exactly at zero energy and are separated from other ordinary quasi-particle excitations by a finite energy gap. For this reason, Majorana bound states are also often referred to as Majorana zero modes (MZMs). It has been shown that MZMs in a $2D$ material obey quantum exchange statistics, which are neither fermionic nor bosonic [11–13]. MZMs are supposed to be an example of so-called non-Abelian anions. This means that the replacement of two MZMs implements a non-trivial rotation of the degenerate subspace of the ground state, while subsequent rotations do not necessarily commute. This property makes non-Abelian anions such as MZMs promising potential building blocks for topological quantum computers, where logic gates would then be performed by exchanging anyons [17, 18]. In a broad sense, two MFs together give a Dirac's fermion [20].

Dirac's points lie at the heart of many fascinating phenomena in condensed-matter physics: from massless electrons in graphene to the emergence of conducting edge states in topological insulators. At a Dirac's point, two energy bands intersect linearly and the particles behave as relativistic Dirac's fermions. In solids, the rigid structure of the material sets the mass and velocity of the particles, as well as their interactions. A different, highly-flexible approach is to create model systems using fermionic atoms trapped in the periodic potential of interfering laser beams—a method, which so far has only been applied to explore simple-lattice structures. In Ref. [21], it was reported about the creating, moving and merging Dirac's points with a Fermi gas in a tuneable optical honeycomb lattice. Using momentum-resolved interband transitions, it was observed a minimum band gap inside the Brillouin zone at the position of the Dirac's points. We exploit the unique tunability of lattice potential to adjust the effective mass of the Dirac's fermions by breaking inversion symmetry. Moreover, changing the lattice anisotropy allows to move the position of the Dirac points inside the Brillouin zone. When increasing the anisotropy beyond a critical limit, the two Dirac's points merge and annihilate each other—a situation, which has recently attracted considerable theoretical interest [22–26], but seems extremely challenging to observe in solids [27]. This topological transition was investigated in the lattice parameter space, and it was found excellent agreement with *ab initio* calculations. These results not only pave the way to model materials where the topology of the band structure plays a crucial role, but also provide an avenue to explore many-body phases resulting from the interplay of complex-lattice geometries with interactions [28, 29].

According to Ref. [30], when electrons are confined in a two-dimensional ($2D$) system, typical quantum-mechanical phenomena such as Landau quantization can be detected. Graphene systems, including the single atomic layer and few-layer stacked crystals, are ideal $2D$ materials for studying a variety of quantum-mechanical problems. In this article, the experimental progress in the unusual Landau quantized behaviours of Dirac's fermions in monolayer and multilayer graphene by using scanning tunnelling microscopy (STM) and scanning tunnelling spectroscopy (STS) was considered. Through STS measurement of the strong magnetic fields, distinct Landau-level spectra and rich level-splitting phenomena are observed in different graphene layers. These unique properties provide an effective method for identifying the number of layers, as well as the stacking orders, and investigating fundamentally the physical phenomena for graphene. Moreover, in the presence of a strain and charged defects, the Landau quantization of graphene can be significantly modified, leading to unusual spectroscopic and electronic properties.

Graphene—a one-atom-thick film—consists of a honeycomb-like hexagonal carbon lattice that exhibits a truly two-dimensional ($2D$) nature [31–33]. Two equivalent carbon atoms—A and B—exist in each unit cell of the hexagonal lattice in graphene. These atoms are strongly connected by in-plane covalent bonds, *i.e.*, the σ bond is hybridized between one s orbital and two p orbitals [34, 35]. The remaining unhybridized p orbital of each atom, which is perpendicular to the planar surface, can support the fourth valence electron of carbon, leading to the formation of a π-electronic band. The electrons filling the π bands can move freely in the graphene plane, as if they are relativistic particles [36, 37]. These π-electronic states are responsible for the electronic properties of graphene at low energies, whereas the σ-electronic states form energy bands far from the Fermi energy. In the low-energy range, the freely moving π electrons in graphene are described by the Dirac's equation [38, 39], rather than the usual Schrödinger equation, because of the two-sublattice crystal structure. The electronic hopping between the neighbouring sublattices leads to the formation of a conical energy spectrum, with the valence band and the conduction band touching at a point-like Fermi surface called the Dirac's point, which yields two inequivalent points K and K' at the corners of the hexagonal Brillouin zone. As a result, the quasi-particles in graphene exhibit the linear dispersion relationship $E = \hbar k v_F$ in the vicinity of the Dirac's point, where the carriers behave as massless fermions that mimic the physics of quantum electrodynamics with a constant Fermi velocity $v_F \approx 10^6$ m/s [40, 41].

2. MATERIAL AND METHODS

Samples of self-doped $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ and $SmMnO_{3+\delta}$ ($\delta \cong 0.1$) manganites were obtained from high-purity oxides of samarium and electrolytic manganese taken in a stoichiometric ratio. The synthesized powder was pressed under pressure of 10 kbar into discs of 6 mm in diameter, 1.2 mm thick and sintered in air at a temperature of 1170°C for 20 h followed by cooling at a rate of 70°C/h. The resulting tablets were a single-phase ceramic according to x-ray data. X-ray studies were carried out at 300 K on DRON-1.5 diffractometer in radiation Ni$K_{\alpha1+\alpha2}$. Symmetry and crystal-lattice parameters were determined by the position and character splitting reflections of the pseudocubic lattice of perovskite type. Temperature and field dependences of dc magnetization were measured using a VSM EGG (Princeton Applied Research) vibrating magnetometer and a nonindustrial magnetometer in ZFC and FC modes.

3. EXPERIMENTAL RESULTS AND DISCUSSION

3.1. Majorana and Dirac's Fermion Excitations in $La_{0.15}Sm_{0.85}MnO_{3\ \delta}$ Controlled by External Magnetic Field

As can be seen in Fig. 1, the temperature dependence of the 'supermagnetization' $M(T)$ in the first Landau zone of $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ in the magnetic field of 100 Oe has the shape of a Dirac's cone-like truncated hill with a flat top near the average temperature $T \cong 4.6$ K, which corresponds to the excitation of two coupled Majorana zero modes arising on spatially separated point defects. In a broad sense, two MFs together give a Dirac's fermion. According to Fig. 2, in external magnetic field with $H = 350$ Oe, a distinct magnetic response appears in the first Landau band in the shape of two weak spiky peaks near the average temperature near the average temperature $T_{MZM} \cong 4.6$ K, which

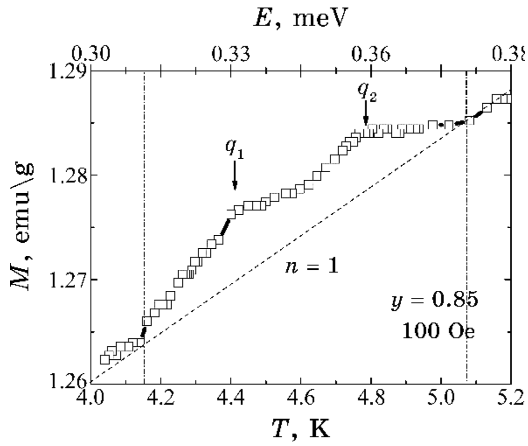

Fig. 1. The thermal excitation of two coupled Majorana zero modes in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ in the Landau band with $n = 1$ with energy $E_{MZM} \cong 0.35$ meV of the shape of $2D$ Dirac's cone-like truncated hill with a flat top near the average temperature $T_{MZM} \cong 4.6$ K in external magnetic field with $H = 100$ Oe.

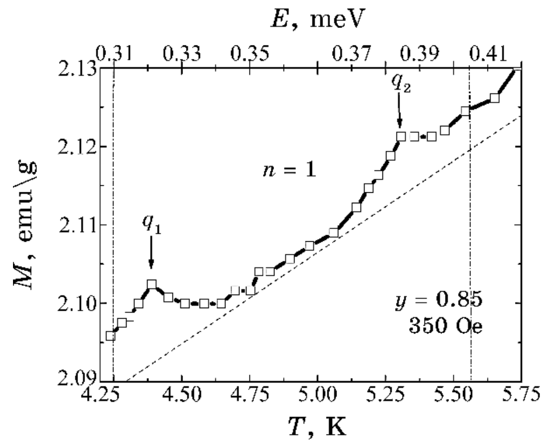

Fig. 2. The thermal excitation of spiky features in the magnetic response of $M(T)$ in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ in the Landau band with $n = 1$ arise from excited states containing either only static magnetic fluxes and no mobile fermions, or from excited states, in which fermions are closely coupled to fluxes in the external magnetic field with $H = 350$ Oe.

corresponds to the excitation of two coupled Majorana zero modes arising on spatially separated point defects. These spiky features in the magnetic response arise from excited states containing either both only static magnetic fluxes and no mobile fermions, or from excited states, in which fermions are closely coupled to fluxes.

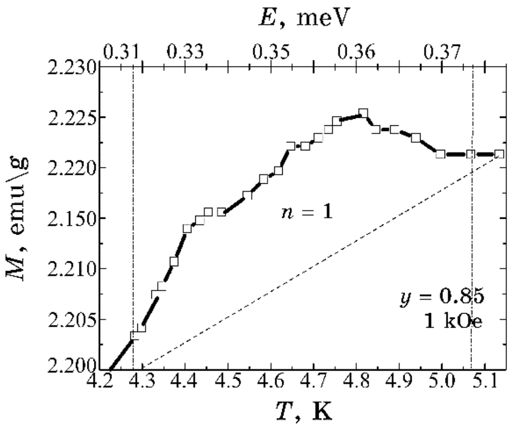

Fig. 3. The thermal excitation of two coupled Majorana zero modes in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ in the Landau band with $n = 1$ with energy $E_{MZM} \cong 0.35$ meV of the shape of 2*D* Dirac's cone-like truncated hill with a flat top near the average temperature $T_{MZM} \cong 4.6$ K in external magnetic field with $H = 1$ kOe.

The structural factor is significantly different in the Abelian and non-Abelian quantum spin liquids (QSLs). Coupled fermion-flow composites appear only in the non-Abelian phase. The main feature of the dynamical structure factor at the isotropic point of the non-Abelian phase is the presence of a pointed δ-component caused by Majorana fermions coupled to flow pairs and a broad hump-like component caused by fermion continuum excitation. Founded Dirac's cones' features of the 'supermagnetization' $M(T)$ in the 1st Landau zone are direct evidence of the excitation of Dirac's fermions in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$. An alternate permutation of the spiky double peaks and Dirac's cones' features of the magnetization $M(T)$ of $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ may be explained by the existence in this material of two hidden states of the chiral spin liquid.

A further increase in the external magnetic-field strength to the value $H = 1$ kOe led to the formation in the Landau zone with $n = 1$ of intensive truncated hill feature with a flat top near the average temperature $T \cong 4.6$ K in the magnetic response (Fig. 3). In external magnetic field with $H = 3.5$ kOe, only the step-like quantum oscillations of temperature dependences of 'supermagnetization' of incompressible quantum spinon liquid were found (Fig. 4).

3.2. Majorana and Dirac's Fermion Excitations in $SmMnO_{3\ \delta}$ Controlled by External Magnetic Field

Previously, the formation of a broad continuum of spinon pair excitations in $SmMnO_{3+\delta}$ in the 'weak magnetic field' regime with $H = 100$

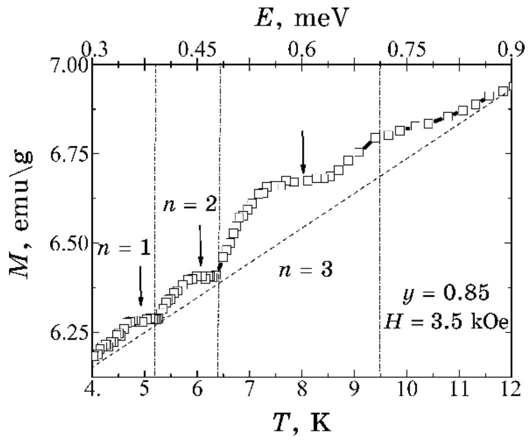

Fig. 4. With an increase in the field strength to $H = 3.5$ kOe, a step-like quantization of the spinon spectrum of $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ appeared in the form of the formation of threshold features of magnetization in the Landau bands $n = 1$, $n = 2$, and $n = 3$ in the kind of steps (plateaus) corresponding to the integer filling of three Landau bands with a finite gap by spinons. As the temperature rises, the height of the thresholds and the width of the steps increase. Emergence of new quantum oscillations of the magnetization–temperature dependences in the strong field regime of measurements is caused by the Landau quantization by the gauge field of the spinon gas with a fractional spin $S = 1/2$, moving on circular orbits in the direction transverse to the direction of the 'magnetic' component, **B**, of the gauge field.

Oe, 1 kOe in the FC mode is explained within the framework of the Landau quantization models of the compressible spinon gas with fractional values of the factor ν filling three overlapping bands with quantum numbers $n = 1$, 2, and 3 [42]. In the regime of 'strong magnetic field' with $H = 3.5$ kOe, the new step-like quantum oscillations of temperature dependences of supermagnetization of incompressible spinon liquid were found. According to the experimental results obtained in this work, in low-energy Landau zone with number $n = 1$, with an increase in the strength of the measuring field H, two alternating supermagnetization features are formed, which are characteristic of the excitation of $2D$ Majorana and Dirac's fermions in frustrated manganites [43].

In this work, a singularity of supermagnetization $M(T)$ of $SmMnO_{3+\delta}$ in Landau band with number $n = 1$ in a magnetic field with $H = 100$ Oe has a double-peak shape with a strong dip near the average temperature $T \cong 4.65$ K (Fig. 5). In a magnetic field with $H = 350$ Oe, a singularity of $M(T)$ in Landau band with $n = 1$ has a shape a truncated Dirac's cone with a flat top in a wider energy range $\Delta E \cong 0.05$ meV with a weak dip in its top near the average excitation temperature of massless

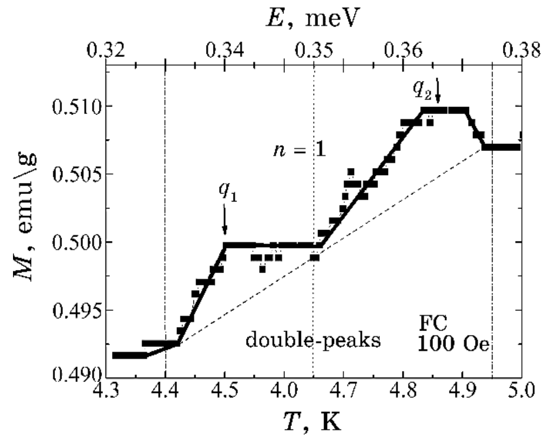

Fig. 5. An alternate excitation in the FC measurement mode for $SmMnO_{3+\delta}$ in the first Landau band with $n = 1$ of spiky double-peaks' features in the magnetic response arise from excited states containing either only static magnetic fluxes and no mobile fermions, or from excited states, in which fermions are closely coupled to fluxes near average temperature $T \cong 4.65$ K in external magnetic field with $H = 100$ Oe (CSL1 state).

Dirac's quasi-particles, namely, $T \cong 4.6$ K (Fig. 6). It can be seen in Fig. 7, this regularity in the alternation of $M(T)$ features in low-energy Landau band with a magnetic field rise is also preserved in the measuring field with $H = 1$ kOe, despite a significant expansion of $M(T)$ features caused by the appearance of strong fluctuations of the topological order in spin system: in the $n = 1$ zone, a two-peak supermagnetization feature forms near the average temperature $T \cong 4.6$ K. As clearly shown in Fig. 8, a characteristic feature of the 'supermagnetization' oscillating in the temperature range 5–8 K in a 'strong' magnetic field with $H = 3.5$ kOe is the appearance of periodic threshold step-like features of width $\Delta E \cong 0.08$–0.15 meV with characteristic narrow plateaus in the temperature dependence of the sample magnetization in Landau band with $n = 1$. Thus, an increase in the magnetic-field strength in $SmMnO_{3+\delta}$ in the 'weak magnetic fields' mode is also accompanied by a alternate rearrangement of singularities of supermagnetization in low-energy Landau band with number $n = 1$ as well as in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$.

According to Ref. [45], the experimental discovery in 1982, in a two-dimensional electron system, of quantized Hall plateaus with Hall conductivity $\sigma_{xy} = \nu e^2/h$ showing fractional values $\nu = 1/3$ and $\nu = 2/3$, marked the beginning of one of the most surprising and far-reaching developments in condensed-matter physics in the second half of the twentieth century. These fractional quantized Hall (FQH) plateaus, together with plateaus at other rational fractional values of ν, were

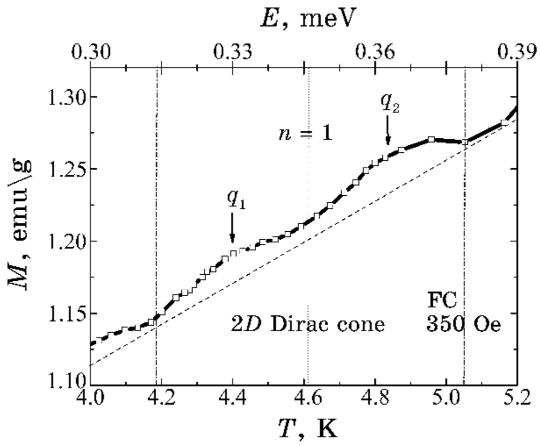

Fig. 6. An alternate excitation in the FC measurement mode in $SmMnO_{3+\delta}$ in the first Landau band with $n = 1$ of two coupled Majorana zero modes with energy $E_{MZM} \cong 0.35$ meV of the form of $2D$ Dirac's cones-like feature near the average temperature $T_{MZM} \cong 4.6$ K of the temperature dependence of the magnetization in external magnetic field with $H = 350$ Oe (CSL2 state).

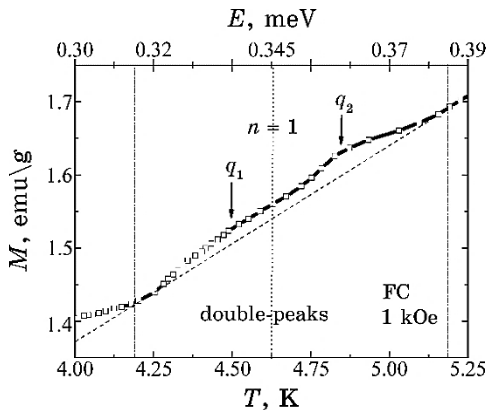

Fig. 7. An alternate excitation in the FC measurement mode in $SmMnO_{3+\delta}$ in the first Landau band with $n = 1$ of two coupled Majorana zero modes with energy $E_{MZM} \cong 0.35$ meV of the form of double-peaks' features with $T_{MZM} \cong 4.6$ K of the temperature dependence of the magnetization in external magnetic field with $H = 1$ kOe (CSL1 state).

understood to be manifestations of a new type of correlated electron state, with a number of peculiar properties.

Continuing experimental and theoretical efforts have revealed a wide variety of FQH states, as well as other unusual phenomena, which can occur in two-dimensional electron systems in a magnetic field at

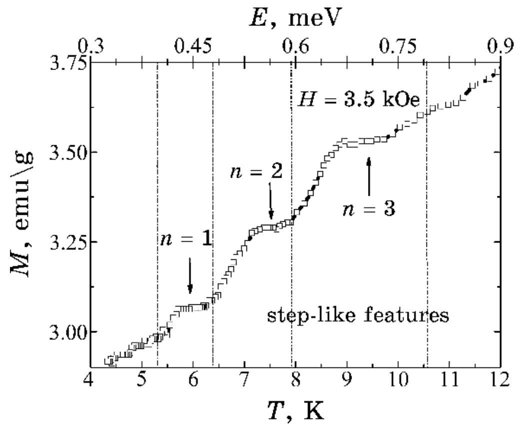

Fig. 8. With an increase in the field strength to $H = 3.5$ kOe, a step-like quantization of the spinon spectrum appeared in $SmMnO_{3+\delta}$ of the form of the formation of threshold features of magnetization in the Landau bands with $n = 1$, $n = 2$, and $n = 3$ in the shape of steps (plateaus) corresponding to the integer filling of three Landau bands with a finite gap by spinons. As the temperature rises, the height of the thresholds and the width of the steps increase. Emergence of new quantum oscillations of the magnetization–temperature dependences in the strong-field regime of measurements is caused by the Landau quantization by the gauge field of the spinon gas with a fractional spin $S = 1/2$, moving on circular orbits in the direction transverse to the direction of the 'magnetic' component, **B**, of the gauge field.

low temperatures, in different materials and under different conditions. Indeed, experiments on these systems continue to produce surprises, and the field of quantum Hall effects remains a vital area of condensed-matter research today. Moreover, insights gained from the exploration of FQH states have also inspired predictions of a variety of other unusual states in other systems. One peculiar feature of the FQH states, which was understood quite early, is that they must necessarily have well-defined charged excitations (quasi-particles) with a charge that is a fraction of the electronic charge. It was also predicted that collections of these quasi-particles should obey fractional statistics, such that the effective wave function for the quasi-particles would be multiplied by a complex phase factor, when two quasi-particles are interchanged, in contrast with the factor of ±1 obtained on interchange of the familiar bosons or fermions. The existence of quasi-particles with fractional charge and statistics is essentially inescapable logical consequence of the existence of fractional quantized Hall states.

Thus, in one sense, the observation of an FQH plateau might be considered as a direct demonstration of the existence, in principle, of quasi-particles with fractional charge and statistics.

An alternate permutation of the spiky double peaks and truncated-hill features of the magnetization $M(T)$ in $SmMnO_{3+\delta}$ may be explained by the existence in this material of two well-known in the literature hidden states CSL1 and CSL2 of the chiral spin liquid (CSL) [44]. The chiral spin liquid state spontaneously breaks time reversal symmetry, but retains other symmetries. There are two topologically different CSLs separated by a quantum critical point. As shown in Ref. [43], the chirality of spins corresponds to the flux of the internal gauge field within the framework of the gauge field theory, which is often used to describe a spin liquid. According to the proposed models, an external magnetic field induces a static internal flux in a system of spins with a four-spin exchange interaction. A quantitative assessment of the effect made it possible to establish that the magnitude of the resulting magnetic orbital field acting on the spinons is comparable to exceed the magnitude of the external magnetic field. It is argued that, due to the fact that the rigidity of the internal gauge field is very small, the homogeneous state of the spinon–gauge field system is unstable at low temperatures due to strong Landau quantization of the spinon energy spectrum. This instability resembles the situation in metals in the strong magnetic-field regime, but, according to the quantitative assessment, the range of temperatures and fields of existence of instability of the homogeneous state in the spinon–gauge field system is significantly wider. The response of the spinon–gauge field system changes dramatically at low temperatures: the Landau quantization of spinons in a static internal field at temperatures below the critical value is not smeared by temperature.

A similar phenomenon has been well studied previously for magnetic oscillations in ordinary metals. The instability regime of a homogeneous state in the spinon–gauge field system is much broader than in ordinary metals, because the rigidity of the internal gauge magnetic field is much smaller than that of the external field. Roughly speaking, in the spinon–gauge field system, spinon states with integer filling of Landau levels are more stable than states with continuously changing filling. Therefore, it is energetically favourable for the internal gauge field to adjust its discreteness to achieve this. Such instability allows direct investigation of the properties of the spinon Fermi surface, using the results of an experimental study of the oscillations of the magnetization of the sample. Two-type features of the magnetization $M(T)$ in $SmMnO_{3+\delta}$, measured in the temperature interval 4.2–6 K in the FC mode, also confirm our assumption about the existence of two types of $2D$ fractional excitons in this material.

5. CONCLUSION

As can be seen in the temperature dependence of the 'supermagnetization' $M(T)$ in the first Landau zone of $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ in the magnetic field 100 Oe has the shape of a Dirac's cone-like truncated hill with a flat top near the average temperature $T \cong 4.6$ K, which corresponds to the excitation of two coupled Majorana zero modes arising on spatially separated point defects. In a broad sense, two MFs together give a Dirac's fermion. In external magnetic field with $H = 350$ Oe, a distinct magnetic response appears in the first Landau band in the shape of two weak spiky peaks near the average temperature near the average temperature $T_{MZM} \cong 4.6$ K. These spiky features in the magnetic response arise from excited states containing either both only static magnetic fluxes and no mobile fermions, or from excited states, in which fermions are closely coupled to fluxes. The structural factor is significantly different in the Abelian and non-Abelian QSLs. Coupled fermion-flow composites appear only in the non-Abelian phase. The main feature of the dynamical structure factor at the isotropic point of the non-Abelian phase is the presence of a pointed δ-component caused by Majorana fermions coupled to flow pairs and a broad hump-like component caused by fermion continuum excitation. Founded Dirac's cones features of the 'supermagnetization' $M(T)$ in the first Landau zone are direct evidence of the excitation of Dirac's fermions in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$. An alternate permutation of the spiky double peaks and Dirac's cones features of the magnetization $M(T)$ in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ may be explained by the existence in this material of two hidden states of the chiral spin liquid. A further increase in the external magnetic-field strength to the value $H = 1$ kOe led to the formation in the Landau zone with $n = 1$ of intensive truncated-hill feature with a flat top near the average temperature $T \cong 4.6$ K in the magnetic response. In external magnetic field with $H = 3.5$ kOe, only the step-like quantum oscillations of temperature dependences of 'supermagnetization' of incompressible quantum spinon liquid were found.

The condensed-matter version of MFs has attracted theoretical interest, mainly because of their special exchange statistics. They are non-Abelian anyons, meaning that particle exchanges are non-trivial operations, which in general do not commute. This is unlike other known particle types, where an exchange operation merely has the effect of multiplying the wave function with +1 (for bosons) or −1 (for fermions) or a general phase factor ϕ (for 'ordinary' Abelian anyons). Furthermore, an MF is in a sense half of a normal fermion, meaning that a fermionic state is obtained as a superposition of two MFs. In a broad sense, two marjorams together give a Dirac's fermion. Normally, this is a purely mathematical operation without physical consequences, since the two MFs are spatially localized close to each other; overlap is significant and cannot be addressed individually. When we talk about MFs here, we mean that a fermionic state can be written as a superposition of two MFs, which are spatially separated. Such a highly delocalized fermionic state is protected from most types of decoherence, since it cannot be changed by local perturbations affecting only one of its Majorana constituents. The state can, however, be manipulated by physical exchange of MFs because of their non-Abelian statistics, which has led to the idea of low-decoherence topological quantum computation. Electronic properties of materials are commonly described by quasi-particles, which behave as nonrelativistic electrons with a finite mass and obey the Schrödinger equation. We assume that the electron system in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ is similar to graphene systems, where electron transport is essentially governed by the Dirac's equation and charge carriers mimic relativistic particles with zero mass and an effective 'speed of light' $c^* \approx 10^6$ m/s. In the presence of an external magnetic field, the low-energy band structure of a $2D$ electron system develops into discretely dispersionless Landau levels, which are responsible for the Landau quantization of massless Dirac's fermions in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$. In strong magnetic field with $H = 3.5$ kOe, only the step-like quantum oscillations of temperature dependences of 'supermagnetization' of incompressible quantum spinon liquid were found. We assume that the electron system in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ is similar to graphene systems. In the presence of an external magnetic field, the low-energy band structure of a $2D$ electron system develops into discretely dispersionless Landau levels, which are responsible for the Landau quantization of the Majorana and massless $2D$ Dirac's fermions.

In this work, in low-energy Landau zone with number $n = 1$, with an increase in the strength of the measuring field H, two alternating supermagnetization features are formed, which are characteristic of the excitation of $2D$ Majorana and Dirac's fermions in frustrated manganites. A singularity of supermagnetization $M(T)$ of $SmMnO_{3+\delta}$ in Landau band with number $n = 1$ in a magnetic field with $H = 100$ Oe has a double-peak shape with a strong dip near the average temperature $T \cong 4.65$ K. In a magnetic field $H = 350$ Oe, a singularity $M(T)$ in Landau band with $n = 1$ has a shape of truncated Dirac's cone with a flat top in a wider energy range $\Delta E \cong 0.05$ meV with a weak dip in its top near the average excitation temperature of massless Dirac's quasiparticles ($T \cong 4.6$ K). This regularity in the alternation of features $M(T)$ in low-energy Landau band with a magnetic-field rise is also preserved in the measuring field with $H = 1$ kOe despite a significant expansion of $M(T)$ features caused by the appearance of strong fluctuations of the topological order in spin system. It clearly shown, a characteristic feature of the 'supermagnetization' oscillating in the temperature range of 5–11 K in a 'strong' magnetic field with $H = 3.5$ kOe is the appearance of periodic threshold step-like features of width $\Delta E \cong 0.08$–0.15 meV with characteristic narrow plateaus in the temperature dependence of the sample magnetization in Landau band with $n = 1$.

The low-energy band structure of graphene can be approximated as cones located at two inequivalent Brillouin-zone corners. In these cones, the $2D$ energy-dispersion relation is linear and the electron dynamics can be treated as ‘relativistic’, where the Fermi velocity v_F of the graphene plays the role of the speed of light. In particular, at the apex of the cones (termed Dirac’s point), electrons and holes (particles and antiparticles) are degenerate. Landau level (LL) formation for electrons in such system under a perpendicular magnetic field, **B**, has been studied theoretically using an analogy to the $2+1$-dimensional quantum electrodynamics. The exceptionally high mobility in graphene samples allow to investigate transport phenomena in the extreme magnetic quantum limit, such as the fractional quantized Hall (FQH). It was shown the overall positive R_{xy} that indicates that the contribution is mainly from electrons. At high external magnetic fields $|\mathbf{B}| \cong 1-8$ T, $R_{xy}(\mathbf{B})$ exhibits step-like features (plateaus) and R_{xx} is vanishing, which are the hallmark of the QHE. At least, two well-defined plateaus are observed before the QHE features transform into Shubnikov–de Haas oscillations at lower magnetic field. The quantization of R_{xy} for these first two plateaus is better than 1 part in 10^4, precise within the instrumental uncertainty.

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