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Low-Temperature Excitation of 2*D* Majorana Fermion Pairs in $SmMnO_{3+\delta}$ Manganites Controlled by an External Magnetic Field

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In this work, we investigate the evolution of the low-energy spinon-pairs' excitation in the first Landau zone in frustrated $SmMnO_{3+\delta}$ manganites caused by changes in the strength **H** of the measuring field. An alternation of double peaks and Dirac cone of features of the 'supermagnetization' $M(T)$, which are characteristic of two types of excitations of Majorana fermions in hidden topological states CSL1 and CSL2 of chiral quantum spin liquid, are revealed. The strong 'smearing' of features of the magnetization $M(T)$ in $SmMnO_{3+\delta}$ revealed in this work is explained by an increase in quantum fluctuations of the sample magnetization caused by the proximity to the quantum critical point of the magnetic phase diagram of the $La_{1-y}Sm_yMnO_{3+\delta}$ system.

В даній роботі досліджено еволюцію збудження низькоенергетичних спінонних пар у першій зоні Ландау у фрустрованих манганітах $SmMnO_{3+\delta}$, спричинену змінами напружености **H** вимірювального поля. Виявлено чергування подвійних піків і Діраків конус особливостей «надмагнетованости» $M(T)$, характерних для двох типів збуджень Майоранових ферміонів у прихованих топологічних станах CSL1 і CSL2 хіральної квантової спінової рідини. Виявлене в даній роботі сильне «розмивання» особливостей намагнетованости $M(T)$ у $SmMnO_{3+\delta}$ пояснюється збільшенням квантових флюктуацій намагнетованости зразка через близькість до квантової критичної точки магнетної фазової діаграми системи $La_{1-y}Sm_yMnO_{3+\delta}$.

Key words: quantum spin liquid, Majorana zero modes, Dirac semimetal, chiral spin liquid, frustrated manganites.

Ключові слова: квантова спінова рідина, Майоранові нульові моди, Діраків напівметал, хіральна спінова рідина, фрустровані манганіти.

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1. INTRODUCTION

According to Ref. [1], a Majorana fermion (MF) is a particle that is its own antiparticle. In the language of second quantization, this means that $\gamma = \gamma^\dagger$, *i.e.*, the fermionic operator γ squares to 1. The creation and annihilation operators can be written as a superposition of two Majorana operators, $a^\dagger = 1/\sqrt{2}(\gamma_1 + i\gamma_2)$, $a = \sqrt{2}(\gamma_1 - i\gamma_2)$. As such, they also fulfil the commutation identity $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$. The task at hand is to physically separate the two Majorana modes, γ_{2j} and γ_{2j-1}, that make up a single fermionic mode, such that phase errors corresponding to $a^\dagger{}_j a_j = (1 + i\gamma_{2j-1}\gamma_{2j})/2$ are unlikely to occur. Put together, these properties would make the Majorana qubit immune to decoherence. These Majorana's fermions can arise as quasi-particles in superconducting systems, which have been investigated in [1] in a one-dimensional chain first proposed by Kitaev [2]. Shown that, they are bound to zero energy, making them Majorana zero modes—a more apt name given that they no longer obey fermionic statistics—where $[H, \gamma_i] = 0$, with H being the Hamiltonian of the system (more realistically, this condition is relaxed to $[H, \gamma_i] \approx e^{-x/\xi}$ [3], where x is the distance between the Majorana zero modes (MZMs) and ξ is the correlation length of the Hamiltonian). They obey non-abelian statistics that enable the implementation of braid operations. This solves the final piece of the puzzle, where qubit operations are now intrinsically fault-tolerant due to their topological properties.

The so-called Majorana bound states arising on point defects have attracted great interest [4–13]. They can be interpreted as own antiparticles in the sense that, in the language of second quantization, the operator of creation and annihilation of bound states are equal to each other. This means that Majorana bound states carry both zero spin and zero charge. Majorana bound states arise exactly at zero energy and are separated from other ordinary quasi-particle excitations by a finite energy gap. For this reason, Majorana bound states are also often referred to as Majorana zero modes (MZMs). It has been shown that MZMs in a $2D$ material obey quantum exchange statistics, which are neither fermionic nor bosonic [5–7]. MZMs are supposed to be an example of so-called non-Abelian anions. This means that the replacement of two MZMs implements a non-trivial rotation of the degenerate subspace of the ground state, while subsequent rotations do not necessarily commute. This property makes non-Abelian anions such as MZMs promising potential building blocks for topological quantum computers, where logic gates would then be performed by exchanging anyons [11, 12].

According to Ref. [14], in systems with a condensed state, when a quasi-particle is a superposition of electron and hole excitations and its production operator $\gamma^\dagger$ becomes identical to the annihilation operator γ, such a particle can be identified as a Majorana fermion. In the Reed–Green model, the Bogolyubov quasi-particles in the volume become dispersive Majorana fermions, and the bound state formed in the core of the vortex becomes the Majorana zero mode. The former is interesting as a new type of wandering quasi-particles, while the latter is useful as a qubit for topological quantum computing. In condensed matter, the constituent fermions are electrons. Because the electron has a negative charge, it cannot be a Majorana fermion. Nevertheless, Majorana fermions can exist as collective excitations of electrons. The resulting Majorana fermions do not retain the true Lorentz invariance of the Dirac equation, since they do not move at the speed of light. However, with proper length and time scaling, the resulting Majorana fermions also obey the Dirac equation. Such Majorana fermions appear within the boundaries of topological superconductors or in the class of spin-liquid systems. The condensation of bosons in the form of a bound state of Majorana fermions was previously studied in topological superconductors by tunnelling spectroscopy. The tunnelling conductivity spectra of topological superconductors depend on their size and symmetry. In one-dimensional topological superconductors with time reversal violation, there is an isolated single Majorana zero mode at each end.

In this work, a study of the evolution of unusual spiky double—peaks and Dirac cone-like features of the low temperature dependences of 'supermagnetization' $M(T)$ in $SmMnO_{3+\delta}$ samples with increasing external magnetic field strength **H** is carried out. These features, according to numerous literature data, are direct evidence of the excitation of 2*D* Majorana and Dirac fermions in quasi-two-dimensional spin systems.

2. MATERIAL AND METHODS

Samples of self-doped manganites $SmMnO_{3+\delta}$ ($\delta \cong 0.1$) were obtained from high-purity oxides of samarium and electrolytic manganese, taken in a stoichiometric ratio. The synthesized powder was pressed under pressure 10 kbar into discs 6 mm in diameter, 1.2 mm thick and sintered in air at a temperature of 1170°C for 20 h followed by cooling at a rate of 70°C/h. The resulting tablets were a single-phase ceramic according to x-ray data. X-ray studies were carried out with 300 K on DRON-1.5 diffractometer in radiation Ni$K_{\alpha1+\alpha2}$. Symmetry and crystal lattice parameters were determined by the position and character splitting reflections of the pseudo-cubic lattice perovskite type. Temperature dependences of *dc* magnetization were measured using a VSM EGG (Princeton Applied Research) vibrating magnetometer and a nonindustrial magnetometer in FC mode.

3. EXPERIMENTAL RESULTS

As can be seen in Fig. 1, the temperature dependence of the 'supermagnetization' $M(T)$ in the first Landau zone of $SmMnO_{3+\delta}$ in the magnetic field 100 Oe has the shape of two weak spiky peaks near the average temperature $T \cong 4.65$ K. According to Refs [15–17], these spiky features in the magnetic response arise from excited states containing either only static magnetic fluxes and no mobile fermions or from excited states, in which fermions are closely coupled to fluxes. The structural factor is significantly different in the Abelian and non-Abelian QSLs. Coupled fermion-flow composites appear only in the non-Abelian phase. The main feature of the dynamical structure factor at the isotropic point of the non-Abelian phase is the presence of a pointed δ-component caused by Majorana fermions coupled to flow pairs and a broad hump-like component caused by fermion continuum excitation.

According to Figure 2, in external magnetic field $|\mathbf{H}| = 350$ Oe dis-

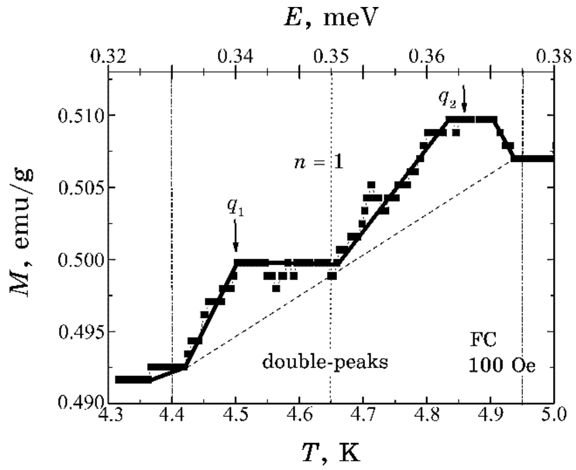

Fig. 1. The thermal excitation of spiky features in the magnetic response $M(T)$ in the Landau band with $n = 1$ arise from excited states containing either only static magnetic fluxes and no mobile fermions, or from excited states in which fermions are closely coupled to fluxes in the external magnetic field $|\mathbf{H}| = 100$ Oe (CSL1 state).

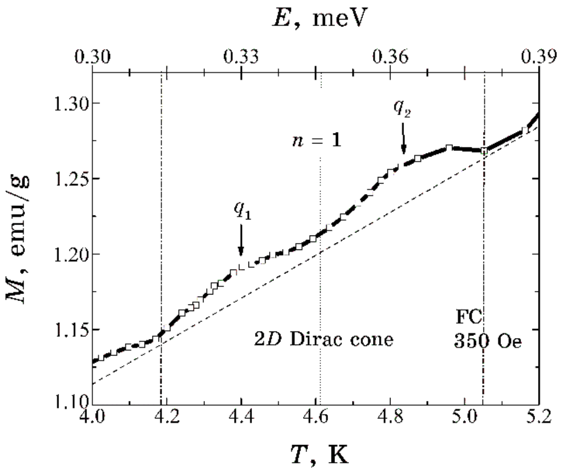

Fig. 2. The thermal excitation of two coupled Majorana zero modes in the Landau band with $n = 1$ with energy $E_{\text{MZM}} \cong 0.35$ meV in the shape of a truncated hill (2*D* Dirac cone) with a flat top near the average temperature $T_{\text{MZM}} \cong 4.6$ K in external magnetic field $|\mathbf{H}| = 350$ Oe (CSL2 state).

tinct magnetic response appears in the first Landau band in the shape of a Dirac cone-like truncated hill with a flat top near the average temperature $T_{\text{MZM}} \cong 4.6$ K, which corresponds to the excitation of two coupled Majorana zero modes arising on spatially separated point defects [4–13], In a broad sense, two MF together give a Dirac fermion [18]. Thus, founded Dirac cones features of the 'supermagnetization' $M(T)$ in the first Landau zone are direct evidence of the excitation of Dirac fermions in $SmMnO_{3+\delta}$. An alternate permutation of the spiky double—peaks and Dirac cones features of the magnetization $M(T)$ in $SmMnO_{3+\delta}$ may be explained by the existence in this material of two hidden states of the chiral spin liquid [1]. A further increase in the external magnetic field strength to the value $|\mathbf{H}| = 1$ kOe led to the formation in the Landau zone with $n = 1$ double-peaks' feature in the magnetic response (Fig. 3). In external magnetic field $|\mathbf{H}| = 3.5$ kOe, only the step-like quantum oscillations of temperature dependences of 'supermagnetization' of incompressible quantum spinon liquid were found (Fig. 4).

4. DISCUSSION

According to Ref. [18], the condensed matter version of MFs has attracted theoretical interest, mainly because of their special exchange statistics. They are non-Abelian anyons, meaning that particle exchanges are non-trivial operations, which, in general, do not

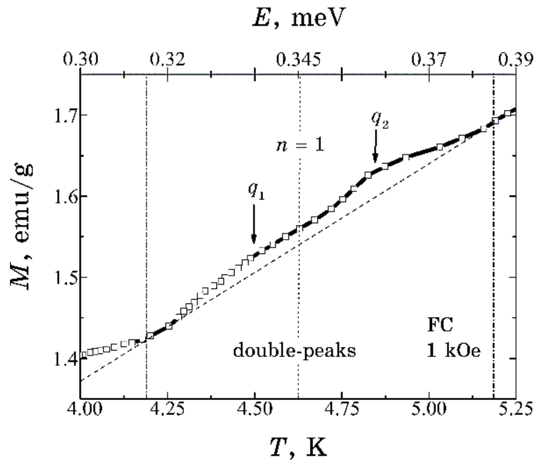

Fig. 3. The thermal excitation of double-peaks feature in the magnetic response $M(T)$ in the Landau band with $n = 1$ arise from excited states containing either only static magnetic fluxes and no mobile fermions, or from excited states, in which fermions are closely coupled to fluxes in external magnetic field $|\mathbf{H}| = 1$ kOe (CSL1 state).

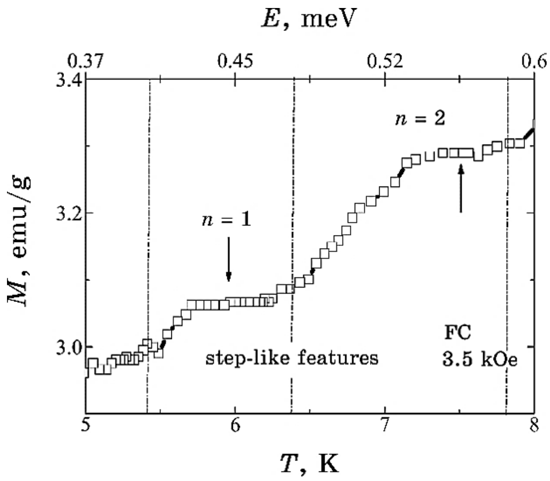

Fig. 4. Ultra-narrow step-like features of the magnetization $M(T)$ in the $SmMnO_{3+\delta}$ in a strong magnetic field $|\mathbf{H}| = 3.5$ kOe in the Landau bands with $n = 1$ and 2. With growth of the field up to $|\mathbf{H}| = 3.5$ kOe, there is a transition from a continuous spectrum of thermal excitations of spinon pairs to a discrete one corresponding to an integer filling of three Landau bands with finite gap.

commute.

This is unlike other known particle types, where an exchange operation merely has the effect of multiplying the wave function with +1 (for bosons) or −1 (for fermions) or a general phase factor φ (for 'ordinary' (Abelian) anyons).

Furthermore, an MF is in a sense half of a normal fermion, meaning that a fermionic state is obtained as a superposition of two MFs. In a broad sense, two marjorams together give a Dirac fermion. It should be noted, however, that any fermion could be written as a combination of two MFs, which basically corresponds to splitting the fermion into a real and an imaginary part, each of which is an MF. Normally, this is a purely mathematical operation without physical consequences, since the two MFs are spatially localized close to each other; overlap is significant and cannot be addressed individually. When we talk about MFs here, we mean that a fermionic state can be written as a superposition of two MFs, which are spatially separated. Such a highly delocalized fermionic state is protected from most types of decoherence, since it cannot be changed by local perturbations affecting only one of its Majorana constituents. The state can, however, be manipulated by physical exchange of MFs because of their non-Abelian statistics, which has led to the idea of low-decoherence topological quantum computation. Being its own hole means that an MF must be an equal superposition of an electron and a hole state. It is natural to search for such excitations in superconducting systems, where the wave functions of Bogolyubov quasi-particles have both an electron and a hole component.

To provide explanation for the rarity of 2*D* Dirac materials as well as clues in searching for new Dirac systems, authors [19] review the recent theoretical aspects of various 2*D* Dirac materials, including graphene, silicene (silicone), germanene, graphynes, several boron and carbon sheets, transition metal oxides and artificial lattices (electron gases and ultracold atoms). As shown, the Dirac cones are rather robust under perturbation. For example, when a uniaxial or shear strain is applied, the band structure of graphene keeps gapless and the Dirac point moves to a new $\mathbf{k}$ location near the original one. At present, Dirac cone merging is achieved only in artificial honeycomb lattices where parameters are much more adjustable. By patterning CO molecules on clean Cu(111), the hexagonal potential lattice of electron gases was effectively modulated to demonstrate a transition from massless to massive Dirac fermions in the system. In an ultracold gas of 40 K atoms trapped in a 2*D* honeycomb optical potential lattice, the merging and annihilation of two Dirac points were clearly recorded, when the lattice anisotropy exceeds a critical limit. A simplified analysis on a general 2*D* system, which contains two atoms of the same species in each unit cell, demonstrated that a hexagonal cell is the most favourable for the existence of Dirac cones, and the favourableness is gradually diminished, when the cell evolves into a square one. According to Ref. [19], to achieve Dirac materials, at least, three conditions are required.

(1) Symmetry. Specific symmetries are required to reduce the number of equations to be solved. The **k**-points should be unchanged after the symmetry operation (invariant **k**-point). Too low or too high symmetries are both disadvantageous.

(2) Parameters. Even when the number of equations is equal to the number of variables, the solution is not necessarily exist, since the variables (k_x and k_y) are real numbers and appear in the equations in the form of a sine or cosine function. Therefore, proper parameters are required. This is usually described as a phase diagram in the parameter space.

(3) Fermi level and band overlap. The Fermi level should lie at the Dirac points and there should not have any other band than Dirac points overlap at the Fermi level.

In Ref. [20], an ultracold Fermi gas of 40 K atoms in a two-dimensional tuneable optical lattice was investigated, which can be continuously adjusted to create square, triangular, dimer and honeycomb structures. It was exploited the momentum resolution of the interband transitions to directly observe the movement of the Dirac points. Starting from a honeycomb lattice, the authors gradually increase the tunnelling along the x direction by decreasing the intensity of the x-beam. The position of the Dirac points continuously approaches the corners of the Brillouin zone, as expected from an *ab initio* two-dimensional band structure calculation. When reaching the corners, the two Dirac points merge, annihilating each other. There, the dispersion relation becomes quadratic along the q_y axis, remaining linear along q_x. Beyond this critical point, a finite band gap appears for all quasi-momenta of the Brillouin zone. This situation signals the transition between band structures of two different topologies, one containing two Dirac points and the other none. In Ref. [21], it was shown that the introduction of lattice anisotropy causes Dirac cones to shift in response to the applied strain, leaving a pseudo-gap at the original Dirac points. Here, a group-theory analysis is combined with first-principles calculations to reveal the movement characteristics of Dirac points and band gaps in various graphynes under rotating uniaxial and shear strains. Graphene, where linear effects dominate, is different from α-, β-, and γ-graphynes, which generate strong nonlinear responses due to their bendable acetylenic linkages. However, the linear components of the electronic response, which are essential in determining material performance such as intrinsic carrier mobility due to electron–phonon coupling, can be readily separated, and are well described by a unified theory. The movement of the Dirac points in α-graphyne is circular under a rotating strain, and the pseudo-gap opening is isotropic with a magnitude of only 2% that in graphene. According to Ref. [22], there has been a recent growing interest for various physical systems exhibiting a multiband excitation spectrum with crossing points between the bands. This interest was boosted by the discovery of graphene, where the low energy spectrum is described by a 2*D* Dirac equation for massless fermions, giving the name 'Dirac point' to such linear crossing point. In two dimensions, a band touching is a topological defect protected by time-reversal and inversion symmetries. Such a contact point is characterized by a winding number w, which describes the winding of the phase of the wave function when moving around this point in reciprocal space. Such singularities may emerge or disappear under variation of external parameters under the constraint that the sum of their winding numbers is conserved.

In Ref. [23], authors propose a simple Hamiltonian H_{eff} to describe the motion and the merging of Dirac points in the electronic spectrum of two-dimensional electrons. This merging is a topological transition, which separates a semi-metallic phase with two Dirac cones from an insulating phase with a gap. They calculated the density of states and the specific heat. The spectrum in a magnetic field **B** is related to the resolution of a Schrödinger equation in a double well potential. The effective Hamiltonian H_{eff} has the remarkable property to describe continuously the Landau level spectrum from the $\in_n \propto \sqrt{n|\mathbf{B}|}$ dependence with double degeneracy for well-separated Dirac cones to the $\in_n \propto (n+1/2)|\mathbf{B}|$ usual dependence for a massive particle. For negative parameter $\delta \propto \Delta/|\mathbf{B}|^{2/3}$, the problem is similar to the one of a particle in a double well potential. In the limit of large negative δ that is far from the transition or in a weak magnetic field, the potential has two well-separated valleys, which are almost uncoupled. This corresponds to the situation of two independent valleys. Note that in this limit the energy shift between the two valleys is $2\sqrt{\delta}$. When δ diminishes, we progressively increase the coupling between valleys. The degeneracy of Landau levels is progressively lifted. They evolve continuously from a $\sqrt{n|\mathbf{B}|}$ to a linear $(n+1/2)|\mathbf{B}|$ dependence, with a $[(n+1/2)|\mathbf{B}|]^{2/3}$ dependence at the transition. The spectrum in the vicinity of the topological transition is very well described by a semi-classical quantization rule. This model describes continuously the coupling between valleys associated with the two Dirac points, when approaching the transition. It remarkably reproduces the low field part of the Rammal Hofstadter's spectrum for the honeycomb lattice.

In Ref. [24], analysis of the electronic properties of a deformed honeycomb structure arrayed by semiconductor quantum dots (QDs) is conducted theoretically by using tight-binding method. Through the compressive or tensile deformation of the honeycomb lattice, the variation of energy spectrum has been explored. It was shown that, the massless Dirac fermions are generated in this adjustable system and the positions of the Dirac cones as well as slope of the linear dispersions could be manipulated. Furthermore, a clear linear correspondence between the distance of movement d (the distance from the Dirac points to the Brillouin zone corners) and the tuneable bond angle α of the lattice are found in this artificial planar QD structure. These results provide the theoretical basis for manipulating Dirac fermions and should be very helpful for the fabrication and application of high-mobility semiconductor QD devices.

The fact that doped manganites can have the properties of a 2D semimetal has long been known, but relatively little theoretical and experimental work has been done in this interesting area of solid-state physics. The possibility of the existence of a state similar to an exciton dielectric in doped $La_{1-x}Ca_xMnO_3$ manganites was first pointed out in Ref. [25] within the framework of a two-band model of double exchange of charge carriers in the e_g shell of manganese ions without taking into account the electron–phonon interaction. It was shown that nesting of electron and hole regions (pockets) of the Fermi surface, corresponding to two e_g bands of charge carriers, exists in the initial $LaMnO_3$ compound. Because of the nesting of these bands in $LaMnO_3$, charge carriers are unstable to the formation of a gas of electron–hole pairs of the exciton dielectric type. A small dielectric gap appears in the spectrum of quasi-particles, and the system becomes an insulator. The anomalies in the temperature dependences of the *ac*-dielectric permittivity found in the $La_{1-y}Sm_yMnO_{3+\delta}$ system were explained in Ref. [26] in terms of the existing concepts of the Bose–Einstein condensation of an electron–hole liquid in the form of metal drops in an exciton dielectric. In our later work [27], when explaining the effect of external influences on the magnetism of fluctuating low-dimensional electron and spin correlations in frustrated manganites $La_{1-y}Sm_yMnO_{3+\delta}$ ($y = 0.85$, 1.0), we considered several weak coupling models of the appearance of CDW/SDW states and superconductivity, based on the possibility of the existence in some materials of an unusual state—an exciton condensate (EC), in which, according to the literature data, new types of charge/spin density and superconductivity waves can arise under the influence of weak external influences, which are closely related to various types of nesting of electron–hole regions of the Fermi surface and interband (intraband) pairing of charge and spin carriers.

In Ref. [28], a two-band model for the formation of an exciton SDW was constructed taking into account the interband and intraband interactions of charge carriers. The case of complete nesting of electron–hole regions of the Fermi surface is considered for several different values of the wave vector $\mathbf{Q}_i$. It was shown that the change in the shape of the Fermi surface, caused by the relative shift of the conduction and valence bands, leads to competition between different magnetic phases. Within the framework of this model, the static magnetic susceptibility of the electron–hole system was studied, which made it possible to determine the nature of its various instabilities of the exciton SDW type. It is shown that instabilities in the paramagnetic state of a system of EC spins can be strictly identified by studying the peak features of the temperature dependence $\chi(T)$ of their static magnetic susceptibility. When, with a decrease in the temperature of the paramagnetic state of the electron–hole system, it reaches a critical value, then, one can expect the appearance of peaks $\chi(T)$ of various shapes caused by the appearance of SDWs with wave vectors equal to different values of the nesting vector $\mathbf{Q}_i$ of the electron–hole Fermi surface regions. It was found that in the case of exciton (interband) instability, the feature of the paramagnetic susceptibility caused by nesting of the phase transition has the form of a sharp peak, while the intraband instability is characterized by a peak feature spread over a wide range of wave vector values near $\mathbf{Q}_i$. Of particular interest is the appearance of signs of anomalous ferromagnetism in self-doped $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ manganites at temperatures below $T_{FM} \approx 12$ K in magnetic fields $|\mathbf{H}| > 350$ Oe, which we discovered in Ref.[27], apparently associated with the EC ferromagnetism. EC ferromagnetism was first studied theoretically in Refs. [29–31]. A model was considered for the spectrum of fermions ($S = 1/2$), which are unstable to electron–hole pairing in the weak coupling limit. Conditions are found under which singlet ($S = 0$) and triplet ($S = 1$) types of spin pairing can coexist, which leads to an unusual state of a fermion gas of the exciton ferromagnet type. The authors considered three types of fermion gas: 1) semimetals with overlapping phase transitions, 2) semiconductors with a narrow band gap less than the exciton binding energy, 3) metals with very narrow bands. In all three cases, the system is unstable to coherent electron–hole pairing of free charge carriers into a singlet state with a binding energy VC or a triplet state with a binding energy VT, or to the formation of two states that differ in $\mathbf{q}$ vectors.

The energy spectrum of such systems has a semiconductor character with a dielectric gap Δ_C for singlet pairing and Δ_T for triplet pairing. It was shown that the transition of a gas of free charge carriers in such materials to a ferromagnetic ground state is possible even in the case of a weak interaction between charge carriers. It has been established that the simultaneous existence of singlet (Δ_S) and triplet (Δ_T) order parameters is accompanied by the formation of CDWs and SDWs, as well as the removal of spin degeneracy of the electron and hole bands. In this case, if the number of electrons is not equal to the number of holes, the number of charge carriers with opposite spin is different. The total spin of such a system is proportional to the difference in the electron and hole concentrations, which is the reason for the appearance of exciton ferromagnetism at $T \to 0$ K. In a later review [32], it was noted that the weak coupling approximation, in which there is no Hund strong coupling, describes weakly correlated metals well or semiconductors, in which the gluing of electron–hole pairs is carried out by the long-range part of the interaction. This leads to the coexistence of many degenerate exciton states in these materials. Due to this degeneracy, even such a weak exchange interaction as hopping of electron–hole pairs between atoms can play an important role in the formation of exciton ferromagnetism and CDW/SDW of ordered states of fermions [33, 34]. According to Ref. [33], a charge density wave with singlet spin pairing ($S = 0$) and a spin density wave with triplet spin pairing ($S = 1$) can coexist in ECs. In weak coupling models, it is assumed that the main cause of the EC instability is the complete nesting of the electron–hole regions of the Fermi surface. In our opinion, the models of exciton condensate considered above in the simplest case of the weak coupling limit are directly related to the experimental results obtained in this work and can be used as a foundation in our further study of the unusual properties of frustrated manganites of the $La_{1-y}Sm_yMnO_{3+\delta}$ type. The unusual properties of frustrated manganites include the previously discovered coexistence in them at temperatures below 60 K of spin, electron–hole, and superconducting quantum liquids.

4. CONCLUSION

An alternate permutation of the double—peaks and Dirac cone features of the ‘supermagnetization’ $M(T)$ in $SmMnO_{3+\delta}$ during the Landau quantization of the spinon pairs spectrum by the weak external magnetic fields $|\mathbf{H}| = 100$, 350, 1000 Oe may be explained by the existence in this material of two hidden states of the chiral spin liquid CSL1 and CSL2. The double-peaks features in $SmMnO_{3+\delta}$ arise from excited states of the chiral QSL containing from excited states in which Majorana fermions are closely coupled to fluxes. In external magnetic field $|\mathbf{H}| = 350$ Oe magnetic response appears in the first Landau zone in the shape of a Dirac cone feature near the average temperature $T_{MZM} \cong 4.6$ K, which corresponds to the excitation of two coupled Majorana zero modes arising on point defects. In external magnetic field $|\mathbf{H}| = 3.5$ kOe, only the step-like quantum oscillations of temperature dependences of ‘supermagnetization’ of incompressible quantum spinon liquid were found. The strong 'smearing' of the features of the magnetization $M(T)$ in $SmMnO_{3+\delta}$ found in this work is explained by an increase in quantum fluctuations of the sample magnetization caused by the proximity to the quantum critical point of the magnetic phase diagram of the $La_{1-y}Sm_yMnO_{3+\delta}$ system.

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