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Excitation of 2*D* Majorana Fermions in Two Chiral States of a Quantum Spin Liquid in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ Manganites Controlled by External Magnetic Field

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In this work, we investigated the evolution of the low-energy spinon-pairs' excitation in the first Landau zone in frustrated $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ manganites, caused by changes in the strength H of the measuring field. In samples $La_{0.15}Sm_{0.85}MnO_{3+\delta}$, an alternation of spiky double—peaks and truncated-hill—features of 'supermagnetization' $M(T)$, characteristic of two types of excitation of 2*D* Majorana fermions in hidden topological states CSL1 and CSL2 of quantum chiral spin liquid (CSL), was detected.

У даній роботі досліджено еволюцію збудження низькоенергетичних спінонних пар у першій зоні Ландау у фрустрованих манґанітах $La_{0,15}Sm_{0,85}MnO_{3+\delta}$, спричинену зміною напружености H вимірювального поля. У зразках $La_{0,15}Sm_{0,85}MnO_{3+\delta}$ було виявлено чергування загострених подвійних, — піків і зрізаного пагорбу, — особливостей «наднамагнетованости» $M(T)$, характерних для двох типів збуджень 2*D* Майоранових ферміонів у прихованих топологічних станах CSL1 і CSL2 квантової хіральної спінової рідини.

Key words: quantum spin liquid, Majorana zero modes, Dirac semi-metal, chiral spin liquid, frustrated manganites.

Ключові слова: квантова спінова рідина, Майоранові нульові моди, Діраків напівметал, хіральна спінова рідина, фрустровані манґаніти.

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1. INTRODUCTION

The so-called Majorana bound states arising on point defects have attracted great interest [1–11]. They can be interpreted as own antiparticles in the sense that, in the language of second quantization, the operator of creation and annihilation of bound states are equal to each other. This means that Majorana bound states carry both zero spin and zero charge. Majorana bound states arise exactly at zero energy and are separated from other ordinary quasi-particle excitations by a finite energy gap. For this reason, Majorana bound states are also often referred to as Majorana zero modes (MZM). It has been shown that MZMs in a $2D$ material obey quantum exchange statistics, which are neither fermionic nor bosonic [2–8]. MZMs are supposed to be an example of so-called non-Abelian anyons. This means that the replacement of two MZMs implements a non-trivial rotation of the degenerate subspace of the ground state, while subsequent rotations do not necessarily commute. This property makes non-Abelian anyons such as MZM promising potential building blocks for topological quantum computers, where logic gates would then be performed by exchanging anyons [9, 10].

However, not all anyons are useful in topological quantum computing. Only non-Abelian anyons are useful. The simplest realization of a non-Abelian anyon is a quasi-particle or a defect that maintains Majorana's zero mode. Zero mode refers to zero-energy mid-band excitations to which these localized quasi-particles typically correspond in a low-dimensional topological superconductor. This is a real fermionic operator commuting with the Hamiltonian. The existence of such operators guarantees the topological degeneracy of the system. These fermionic operators are real, just as the Majorana version of the Dirac equation is real. However, the key concept here is the non-Abelian anyon, and MZMs are a special mechanism, by which non-Abelian anyons, commonly referred to as Ising anyons, can be generated.

The simplest experimental result confirming the existence of MZM states is a zero-shift tunnelling conductivity peak in a semiconductor (InSb or InAs) nanowire in contact with a conventional metallic superconductor (Al or Nb), which appears only when a finite external magnetic field is applied to the wire.

Anyons can appear in two ways: as localized excitations or as defects in an ordered system. Fractionally charged excitations of Laughlin's quantum Hall fluid are an example of the former. Examples of the latter are the Abrikosov vortices in a topological superconductor. According to Ref. [1], the MZM is a fermionic operator γ quadratic to 1 (and hence necessarily self-adjoint) and commuting with the Hamiltonian H of the system: γ is a fermionic operator, $\gamma^2 = 1$, $[H, \gamma] = 0$. Any operator satisfying the first two conditions is called the Majorana fermionic operator. It is useful to note that propagating Majorana fermions of the neutrino type are supposed to be produced in any superconductor. However, localized

MZMs and their accompanying non-Abelian anyon entanglement are a much more interesting phenomenon. The existence of such operators implies the existence of a degenerate space of ground states in which quantum information can be stored. $2n$ MZM, γ_1, ..., γ_{2n} (they must come in pairs, since each MZM is, in some sense, half a fermion) satisfying the relation $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$, then, the Hamiltonian can be diagonalized simultaneously with the operators $i\gamma_1\gamma_2$, $i\gamma_3\gamma_4$, ..., $i\gamma_{2n-1}\gamma_{2n}$. The ground states can be labelled with ±1 eigenvalues of these n operators, resulting in $2n$-fold degeneracy. Each MZM pair has a two-state system associated with it. This should be contrasted with a set of spin-1/2 particles, for which there is a two-state system associated with each spin. In the case of MZM, we can connect them as we see fit; different pairs correspond to different choices of basis in the $2n$-dimensional ground state Hilbert space.

Unfortunately, the previous mathematics is too idealized for a real physical system. Instead, there may be self-adjoint Majorana fermionic operators γ_1, ..., γ_{2n} satisfying the anticommutation relations $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$ and $[\mathrm{H}, \gamma_j] \sim e^{-x/\xi}$ where x is the length scale, which can be interpreted as the distance between two MZM in a pair, and ξ is the correlation length associated with the Hamiltonian H. In superconducting systems, ξ is the coherence length. All states above the $2n-1$-dimensional low-energy subspace have a minimum energy Δ. For this situation, to approach ideal, it must be possible to make x large enough that the exponent tends to zero quickly. This can happen, if the operators γ_i are localized at the points x_i. Then, γ_i commutates or anticommutates, up to corrections $\sim e^{-y/\xi}$, with all local bosonic or fermionic operators that can be written in terms of electron creation and annihilation operators, supported by the minimum distance y from some point x_i. The effective Hamiltonian for energies much smaller than Δ is the sum of local terms, which means that the products of operators such as $i\gamma_i\gamma_j$ must have exponentially small coefficients. Therefore, the condition $[\mathrm{H}, \gamma_j] \sim e^{-x/\xi}$ is then satisfied. The number of MZM-operators satisfying this condition must be even. Therefore, if we add a term to the Hamiltonian that relates a single zero mode operator to non-zero mode operators, the zero mode operator remains, since, zero modes can only exist in pairs. The exponential 'protection' of MZM, allowing their quantum degeneracy, is provided by an energy gap, which should be as large as possible for the efficient operation of topological quantum computers. In a broad sense, two marjorams together give a Dirac fermion. However, the MZMs must be spaced far apart for topological protection to be applied.

As firstly shown in work [12], the chiral spin liquid (CSL) state spontaneously breaks time reversal symmetry (TRS), but retains other symmetries. There are two topologically different CSLs separated by a quantum critical point. Interestingly, vortex excitations in topologically nontrivial CSLs obey non-Abelian statistics. Kitaev's model [13] on a 'triangular-honeycomb lattice' was studied and it was shown that its exact ground state is a quantum spin liquid (QSL), *i.e.*, a state that spontaneously breaks time reversal symmetry, but not rotational or translational spin symmetry, and has fractional excitations. It is shown that there are two topologically different QSL phases with even and odd Chern numbers, respectively, separated by a quantum phase transition. For a topologically nontrivial (trivial) QSL, vortex excitations obey non-Abelian (Abelian) statistics. The Kitaev's model on this lattice is invariant under time reversal. It is also invariant under lattice inversion and rotation of spins by 180° along the x, y, or z directions, *i.e.*, the spin–orbit interaction lowers the spin symmetry from $SU(2)$ to $D2$. The ground state does not violate either spin symmetries or any translational or point group symmetries of the lattice. The spin correlation function is exactly zero outside of nearest neighbours. However, in both topologically trivial and non-trivial states, the exact ground state spontaneously violates the TRS. As shown by Kitaev, this model can be mapped to a free Majorana model with conserved background Z_2 gauge fields in an enlarged Hilbert space. The physical states are obtained by implementing a set of gauge constraints. For a finite system, the energy difference between ground states with different global fluxes decays exponentially with the linear dimension of the system for gapped Majorana fermions. In the thermodynamic limit, for the Abelian CSL, the ground state degeneracy is fourfold coming from four possible values of the global fluxes, $\Phi_x = \pm 1$ and $\Phi_y = \pm 1$. However, for the non-Abelian CSL the ground state is threefold degenerate because the projection operator annihilates one global ground state. There are six bands $\varepsilon_n(\mathbf{k})$ corresponding to six sites per unit cell. The spectrum is gapped except, when parameter $D(\mathbf{k}) \equiv \mathrm{Det}[\mathrm{H}(\mathbf{k})] = 0$. It is straightforward to show that $D(\mathbf{k}) = 0$, only when $\mathbf{k} = (\pi, -\pi/\sqrt{3})$ and $J' = \sqrt{3}J$. Thus, there is a quantum phase transition between two fully gapped phases, which occurs at $J' = \sqrt{3}J$. These two phases are topologically distinct as can be seen by directly computing the Chern number ν: for $J' > \sqrt{3}J$, $\nu = 0$, while, for $J' < \sqrt{3}J$, $\nu = \pm 1$. That $\nu = \pm 1$ for $J' < \sqrt{3}J$ can be understood as follows: as $J \to \infty$, the spectrum to first order in J' is gapless with Dirac-cones, as for graphene. Terms of second order in J'/J generate an effective Haldane mass term, which breaks TRS, gaps the Dirac cone, and gives $\nu = \pm 1$ [14].

In the usual QHE, the gap at the Fermi level results from the splitting of the spectrum into Landau levels by an external magnetic field. The scenario considered here is different, and involves a $2D$ semi-metal, where there is degeneracy at isolated points in the Brillouin zone between the top of the valence band and the bottom of the conduction band, that is associated with the presence of both inversion symmetry and time-reversal invariance. If inversion symmetry is broken, a gap opens and the system becomes a normal semi-conductor ($\nu = 0$), but, if the gap opens because time-reversal invariance is broken, the system becomes a $\nu = +1$ integer QHE state. If both perturbations are present, their relative strengths determine which type of state is realized. The quantum critical point is at $(J'/J)_c = \sqrt{3}$. For small values of J, the phase is a topologically trivial CSL with Abelian excitations, and for large J, the phase is a topologically nontrivial CSL with vortex excitations obeying Abelian statistics. The spectrum $J \to \infty$ is similar to the Dirac spectrum with a mass gap that vanishes at $J'2/J$. At $J'/J = 2\sqrt{3}/3$, there is a kink, where a jump in $\mathbf{k}$ occurs with minimal energy.

According to Ref. [15], at partial filling of a flat band, strong electronic interactions may favour gapped states harbouring emergent topology with quantized Hall conductivity. Emergent topological states have been found in partially filled Landau levels [16] and Hofstadter bands [17, 18]; in both cases, a large magnetic field is required to engineer the underlying flat band. The recent observation of quantum anomalous Hall (QAH) effects in narrow band moiré systems [19–24] has led to the theoretical prediction that such phases may be realized even at zero magnetic field [22–24]. An observation of Chern insulators at half-integer values of ν suggests spontaneous doubling of the superlattice unit cell, in addition to spin- and valley-ferromagnetism. This is confirmed by Hartree–Fock calculations, which find a topological charge density wave ground state at half filling of the underlying $C = 2$ band, in which the Berry curvature is evenly partitioned between occupied and unoccupied states. It was found the translation symmetry breaking order parameter is evenly distributed across the entire folded superlattice Brillouin zone, suggesting that the system is in the flat band, strongly correlated limit.

These findings show that the interplay of quantum geometry and Coulomb interactions in moiré bands allows for topological phases at fractional superlattice filling that spontaneously break time-reversal symmetry, a prerequisite in pursuit of zero magnetic field phases harbouring fractional statistics as elementary excitations or bound to lattice dislocations. In work [25], there were considered the theoretically predicted and experimentally confirmed topological nodal line semi-metals, focusing, in particular, on the symmetry protection mechanisms of the nodal lines in various materials. Three different protection mechanisms are discussed: a combination of inversion and time-reversal symmetry, mirror reflection symmetry, and non-symmorphic symmetry, and their robustness under the effect of spin–orbit coupling. It was shown that Dirac semimetal is the result of two doubly degenerate bands crossing near the Fermi level at a discrete point in **k**-space, known as a Dirac point. Demanded by both inversion and time-reversal symmetry, the fourfold degenerate Dirac point has a band dispersion that is linearly dependent with **k**. Such a linearly dispersed band structure causes the low energy excitations near the crossing point to be Dirac fermions.

Previously, the formation of a broad continuum of spinon pair excitations in the $La_{1-y}Sm_yMnO_{3+\delta}$ ($y = 0.85$, 1.0, $\delta \cong 0.1$) manganites in the 'weak magnetic field' regime $H = 100$ Oe, 350 Oe, 1 kOe in the FC mode was explained in the framework of the Landau quantization models of the compressible spinon gas with fractional values of the factor ν filling three overlapping bands with quantum numbers $n = 1$, 2, and 3 [26, 27]. In the regime of 'strong magnetic field' $H = 3.5$ kOe, the step-like quantum oscillations of temperature dependences of supermagnetization of incompressible spinon liquid were found.

The aim of this work is to establish the mechanisms of topological phase transitions in frustrated manganites $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ ($\delta \cong 0.1$) induced by an increase of the measuring magnetic field. We investigated the evolution of the low-energy spinon-pairs' excitation in the first Landau zone in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ frustrated manganites caused by changes in the strength H of the measuring field. An alternation of spiky double—peaks and truncated hill—features of 'supermagnetization' $M(T)$, characteristic of two types of excitation of $2D$ Majorana fermions in hidden topological states CSL1 and CSL2 of quantum chiral spin liquid, was detected.

2. MATERIAL AND METHODS

Samples of self-doped manganites $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ ($\delta \cong 0.1$) were obtained from high-purity oxides of lanthanum, samarium and electrolytic manganese, taken in a stoichiometric ratio. The synthesized powder was pressed under pressure 10 kbar into discs of 6 mm in diameter, 1.2 mm thick and sintered in air at a temperature of 1170°C for 20 h followed by cooling at a rate of 70°C/h.

The resulting tablets were a single-phase ceramic according to x-ray data. X-ray studies were carried out with 300 K on DRON-1.5 diffractometer in radiation $\mathrm{Ni}K_{\alpha_1+\alpha_2}$. Symmetry and crystal lattice parameters were determined by the position and character splitting reflections of the pseudocubic lattice perovskite type. Temperature dependences of dc magnetization were measured using a VSM EGG (Princeton Applied Research) vibrating magnetometer and a nonindustrial magnetometer in FC mode.

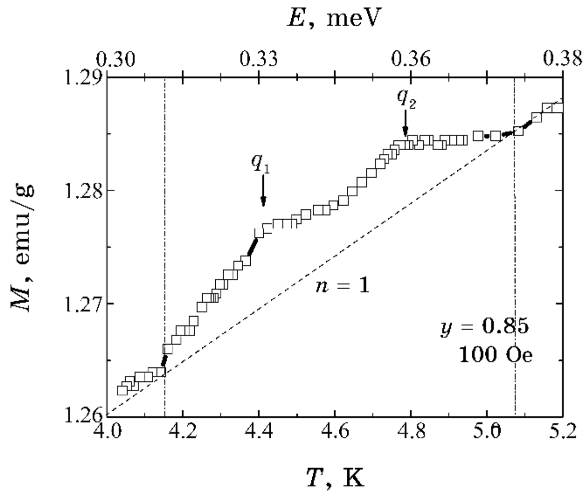

Fig. 1. The thermal excitation of two coupled Majorana zero modes with energy $E_{MZM} \cong 0.35$ meV in the form of a truncated hill with a flat top near the average temperature $T_{MZM} \cong 4.6$ K of the temperature dependence of the magnetization in external magnetic field $H = 100$ Oe (CSL1 state).

3. EXPERIMENTAL RESULTS AND DISCUSSION

In this work, the thermal excitation of two coupled Majorana zero modes with energy $E_{MZM} \cong 0.35$ meV in the form of two wide overlapping peak features of the temperature dependence of the magnetization has been discovered in the first Landau zone with $n = 1$ of the quantized spectrum of spinons in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ in external magnetic field $H = 100$ Oe (Fig. 1). As can be clearly seen from this figure, the temperature dependence of the supermagnetization of the sample $M(T)$ in the first Landau zone in the magnetic field 100 Oe has the shape of a truncated hill with a flat top near the average temperature $T_{MZM} \cong 4.6$ K, which corresponds to the excitation of two coupled Majorana zero modes arising on point defects [1–11]. A slight increase of the magnetic field up to 350 Oe led to a dramatic change in the shape of the temperature dependence of the magnetization of the sample: a singularity of supermagnetization $M(T)$ in a field $H = 350$ Oe has a distinct shape of two weak spiky peaks near the average temperature $T \cong 5$ K (Fig. 2).

According to Refs. [28, 29, 30], these spiky features in the magnetic response arise from excited states containing either only static magnetic fluxes and no mobile fermions, or from excited states in which fermions are closely coupled to fluxes. The structural factor is significantly different in the Abelian and non-Abelian QSLs. Coupled fermion-flow composites appear only in the non-Abelian

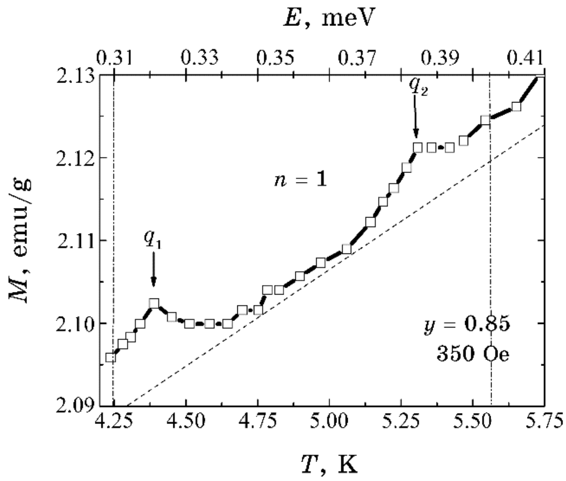

Fig. 2. The thermal excitation of spiky features in the magnetic response arise from excited states containing either only static magnetic fluxes and no mobile fermions, or from excited states, in which fermions are closely coupled to fluxes near average temperature $T \cong 5$ K in external magnetic field $H = 350$ Oe (CSL2 state).

phase. The main feature of the dynamical structure factor at the isotropic point of the non-Abelian phase is the presence of a pointed δ-component caused by Majorana fermions coupled to flow pairs and a broad hump-like component caused by fermion continuum excitation. In external magnetic field $H = 1$ kOe magnetic response appears in the first Landau zone in the shape of a truncated hill with a flat top near the average temperature $T_{MZM} \cong 4.6$ K, which corresponds to the excitation of two coupled Majorana zero modes arising on point defects (Fig. 3).

An alternate permutation of the spiky double—peaks and truncated hill—features of the magnetization $M(T)$ in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ may be explained by the existence in this material of two well-known in the literature hidden states CSL1 and CSL2 of the chiral spin liquid [1]. In external magnetic field $H = 3.5$ kOe, only the step-like quantum oscillations of temperature dependences of supermagnetization of incompressible quantum spinon liquid were found (Fig. 4).

It is of interest to consider in more detail the main results of previous theoretical works devoted to the excitation of low-energy Majorana excitations in two hidden states of a chiral quantum spin liquid. According to Ref. [31], in the classical limit, the dynamic structure factor of a spin system is closely related to the static susceptibility by the simple relation $S(\mathbf{k}) = T\chi(\mathbf{k}, \omega = 0)$. Near the quantum critical point, this relationship is much more complicated. In

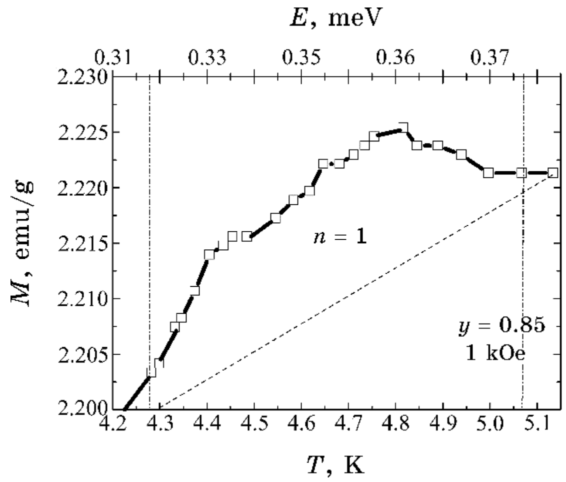

Fig. 3. The thermal excitation of two coupled Majorana zero modes with energy $E_{MZM} \cong 0.35$ meV in the shape of a truncated hill with a flat top near the average temperature $T_{MZM} \cong 4.6$ K in external magnetic field $H = 1$ kOe (CSL1 state).

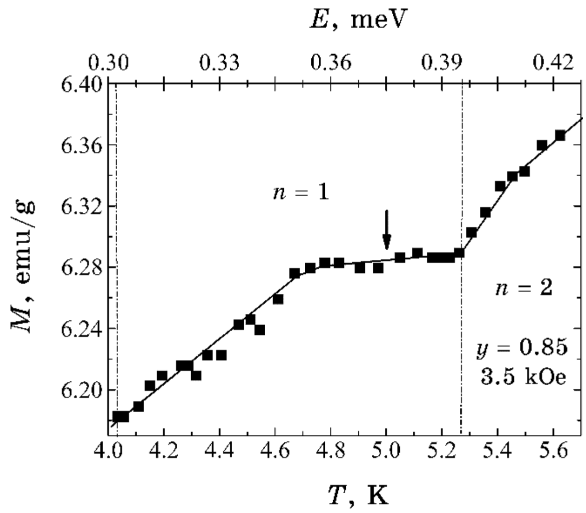

Fig. 4. The thermal excitation of a step-like feature of temperature dependence of supermagnetization of incompressible quantum spinon liquid near average temperature $T \cong 5$ K in external magnetic field $H = 3.5$ kOe.

the absence of structural disorder, static measurements cannot sense fluctuating order.

However, if the disorder is high enough, fluctuations can affect low-energy states. Weak structural disorder creates a low-frequency quasi-elastic part of the spectral function $S(\mathbf{k}, \omega)$. In other words, disorder excludes the spectral gap from consideration, but changes the integral spectral function little. In the presence of weak structural disorder, the function $S(\mathbf{k}, \omega = 0)$ should exhibit dispersion similar to a pure system and therefore can be used to determine the nature of the ordered phase. Thus, the static magnetic susceptibility $\chi(\mathbf{k}, T)$ of a fluctuating spin system is related in the classical limit to the dynamic structure factor $S(\mathbf{k}, \omega = 0)$ by the simple relation $\chi(\mathbf{k}, T) = S(\mathbf{k})/T$. This allows suggest that measurements of static magnetic susceptibility in systems with low structural disorder can be used as a source of information about the integral spectral function of fluctuating spin phases. In the phase with quantum disorder, the presence of any order parameter makes obvious the connection of the system with an external influence that breaks its symmetry. In the case of charge stripes, the Fourier component $V_{\mathbf{k}}$ of the weak potential of the applied external field leads to a linear response $\langle\rho_{\mathbf{k}}\rangle = \chi_{ch}(\mathbf{k})V_{\mathbf{k}} + \ldots$ of a charge stripe system with fluctuating quantum disorder. There is also the question of the reasons for the appearance of density of states waves of one type or another. It is shown that, during the formation of a SDW, here is no difference between the strong coupling limit, in which spin stripes can be considered as a phase separation into metal stripes of excess charge and almost non-conducting AFM bands between them, and the Hartree–Fock description of spin stripes in the weak coupling limit. In the limit of weak coupling, SDW appears because of the opening of small gaps on various segments of the phase transition with a good nesting.

An important consequence of Kitaev's model for QSL [13] is the splitting of electrons into a set of unusual quasi-particles—Majorana fermions, which leads to the appearance in the spectrum of magnetic low-energy excitations of the ground state of chiral spin liquids of several bands of Majorana fermions, the width of which significantly depends on magnetic gauge fluxes and external perturbations. Relatively recently, evidence of this splitting was discovered in α-$RuCl_3$ using neutron scattering techniques, which were explained in [28]. In Ref. [28], a theoretical study of the dynamic structure factor $S^{ab}(\mathbf{q}, \omega)$, a two-dimensional quantum spin liquid in gapless and gap phases was carried out within the framework of the Kitaev's model for a hexagonal spin lattice. The existence of unusual hump-like features of the structure factor caused by the excitation of the continuum of Majorana fermions and the appearance of gauge fluxes is shown. It was found that in both the gap and gapless phases of the QSL, the response disappears at excitation energy values below the gap value Δ, which is directly related to the appearance of the gauge field. A similar gap was found in the modified Kitaev's model in the spectrum of circular excitation. At excitation energies higher than the gap energy Δ, nonzero contributions of fermions to the structure factor $S^{ab}(\omega)$ arise only from excited states with parity opposite to the parity of the ground state. As a result, two excitation modes of Majorana fermions are realized: mode I and mode II. Mode II contains a spiky response, while mode I has no spiky response. In case I, the ground state of the QSL has an odd number of excitations, while, in case II, either there are no magnetic excitations, or their number is even. In mode I, a wide hump-like magnetic response to external excitation is observed, while in case II, the response should be narrow. Responses may overlap or be separate. For case I, the energy-wide response ω is dominated by single-particle excitations, so it manifests itself only within the bandwidth of material fermions.

In Ref. [29], exact results were obtained for the structure factor in the gap and gapless, Abelian and non-Abelian phases of the QL. The structure factor has features when quasi-particles appear in the form of Majorana fermions and Z_2 gauge field flows. In addition to the broad hump-like continuum of fractional spin excitations, the authors found spiky features in the magnetic response. These spiky features arise from excited states containing both only static magnetic fluxes and no mobile fermions, or from excited states, in which fermions are closely coupled to the fluxes. The structure factor is significantly different in Abelian and non-Abelian QSL. Fermion-flow coupled composites appear only in the non-Abelian phase. The main feature of the dynamic structure factor at the isotropic point of the non-Abelian phase ($J_x = J_y = J_z$) is the presence of a pointed δ-component caused by Majorana fermions associated with flux pairs and a broad hump-like component caused by the excitation of the fermion continuum. The beginning of the broad magnetic response feature corresponds to the edge of the Majorana fermion zone. It should be noted that the width of the response of free fermions practically coincides with the width of the calculated density-of-states function $N(\omega)$ of Majorana fermions in the zero-flux phase.

According to Kitaev's model [13], a system of spins with $S = 1/2$ occupy nodes in a hexagonal lattice. The spins interact *via* anisotropic Ising exchange J_a between nearest neighbours, where the three directions designated $a = x, y, z$ select three valence bonds for a given lattice position. Kitaev's model was supplemented in Ref. [29] with a three-spin exchange interaction created by a weak external magnetic field. Circular three-spin exchange interactions break time symmetry and create a gap in the spectrum of Majorana fermions, which leads to an increase in non-Abelian excitations [32]. The first term of the Hamiltonian of the extended Kitaev's model used in Ref. [29] sums up the (nn) Ising exchange interactions in the spin system between two neighbouring i, j nodes of the spin lattice in terms of the Pauli matrices σ_j^a. The additional Hamiltonian term, taken with the coefficient K, describes (nnn) exchange interactions between three spins σ_j^a, σ_j^c, σ_j^b combined with each pair of bonds $\langle ij\rangle_a$ and $\langle jk\rangle_b$ for node j of the crystal lattice. For $K = 0$, two stable phases can exist, known in the literature as gapless and gap Abelian quantum spin liquids. At $K \neq 0$, all these phases acquire a gap and the excitations for the QSL state that were previously gapless become non-Abelian. In all phases, the independent degrees of freedom are Z_2 gauge fluxes located in the cells of the crystal lattice and dynamic Majorana fermions located at lattice sites. The time evolution of Majorana fermions is generated by a Hamiltonian, the shape of which is determined by the particular configuration Z_2 of the gauge field. As noted in [29], different representatives of the family of Kitaev's Hamiltonians correspond to a set of qualitatively different responses. What all these responses have in common is that the spin correlations found are ultrashort [33]: the structure factor contains contributions from correlators only of local or nearest neighbours that have the same spin components. This is a consequence of the static nature of the emerging Z_2 gauge flows.

Only the dependences of the dynamic structure factor $S(\mathbf{q} = \mathbf{0}, \omega)$ on the excitation frequency were presented in the work [29]. In all cases, the dynamic response has a gap, regardless of the existence of a gap in the excitation spectrum of the quantum spin liquid. The minimum gap Δ corresponds to the case of the formation of flow pairs in the QL in adjacent cells of the crystal lattice. The plot of one of the broad components of the magnetic response $S(\mathbf{q} = \mathbf{0}, \omega)$ corresponds to the excitation of Majorana fermions. Its low-energy beginning corresponds to the gap Δ in phase with gapless Majorana excitations, while the end is in the gap Majorana phase. As shown in the work, this broad response occurs when one or two fermions are excited, depending on the spin component and the parameters of the Hamiltonian. In the case of $J_x/J_z = J_y/J_z$, there are only two response components—$S^{zz}(\omega)$ and $S^{xx}(\omega) = S^{yy}(\omega)$. For Abelian states of the QSL, it was found that in the isotropic model the response $S^{aa}(\omega)$ is not equal to zero at an excitation energy higher than the energy Δ required to create flux pairs. This dominant contribution arises from single excitations of Majorana fermions, but the tail extends to higher energies. Although the bandwidth of Majorana fermions mainly determines the magnetic response of a spin system to an external perturbation, the dependence of the response intensity is not related by a simple relation to the fermion density of states function since the response involves the multiplication of Majorana fermions in the presence of flux pairs. However, it reflects such characteristic features of the density function of Majorana states as, for example, the van Hove singularity. According to Ref. [34], the wide hump-like response of neutron scattering by a system of spins in the QL has several different reasons. Firstly, this is scattering, in which any movement of spins with a wave vector $\mathbf{q}=\mathbf{0}$ requires the creation of many magnetic excitations. Secondly, this is not a simple excitation of magnons, since only fractional magnetic excitations are involved in it, which are created simultaneously and thus the scattering process is multiparticle. Thirdly, when describing the low-energy spectrum, quasi-particles do not come first, so the dispersion relation $\omega(\mathbf{q})$ is, in principle, not significant. In the case of a Z_2 spin liquid, in which the triplet excitation decays into a pair of $S = 1/2$ spinons, fractional magnetic excitations are considered as magnetic moments within the quantum dimer model. In order to be observable, each individual spinon must behave like a coherent free particle in the limit of low excitation energies. However, it is still unclear in what energy range such long-lived well-defined fractional particles will exist. The case of symmetrical fragmentation into two particles is too simple a scenario. Apparently, many other quasi-particles additionally arise, which makes it possible to approach cartridge-structures of particles with a wide variety of different crushing schemes. Fractional quasi-particles are not required to have a wide continuous spectrum. Instead, they may form bound or localized states. There is the possibility of deconfinement of quantum chirality at the bicritical point of the phase diagram, which separates two phases with the presence of confinement of quantum spin liquid excitations [35, 36]. Deconfinement of bound states of fractional particles is accompanied by the expenditure of energy required for their decoupling. This does not exclude the possibility of the formation of new discrete composite states with finite binding energy. One of these cases is crushing with a very large excitation asymmetry, in which the flow can be very heavy and inactive, that is, have a very small zone width. Thus, it can interact with an arbitrary number of torques with almost constant energy and thereby produce a conservation of extremely inefficient torques. This leads to broad features of particle excitation, which is typical for photons [36]. The experimental results obtained in this work as well as interpretation of them are in a good agreement with the theoretical predictions discussed above.

4. CONCLUSION

The temperature dependence of the 'supermagnetization' $M(T)$ of $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ in the first Landau zone in the magnetic field 100

Oe has the shape of a truncated hill with a flat top near the average temperature $T_{MZM} \cong 4.6$ K, which corresponds to the excitation of two coupled Majorana zero modes arising on point defects. A slight increase of the magnetic field up to 350 Oe led to a dramatic change in the shape of $M(T)$: a singularity of 'supermagnetization' in a field $H = 350$ Oe has a distinct shape of two weak spiky peaks near the average temperature $T \cong 5$ K. These spiky features in the magnetic response arise from excited states of the chiral QSL containing both only static magnetic fluxes and no mobile fermions or from excited states in which fermions are closely coupled to fluxes. Coupled fermion-flow composites appear only in the non-Abelian phase. In external magnetic field $H = 1$ kOe, magnetic response appears in the first Landau zone in the shape of a truncated hill with a flat top near the average temperature $T_{MZM} \cong 4.6$ K, which corresponds to the excitation of two coupled Majorana zero modes arising on point defects. An alternate permutation of the spiky double—peaks and truncated hill—features of the magnetization $M(T)$ in $La_{0.15}Sm_{0.85}MnO_{3+\delta}$ may be explained by the existence in this material of two well-known in the literature hidden states of the chiral spin liquid. In external magnetic field $H = 3.5$ kOe, only the step-like quantum oscillations of temperature dependences of supermagnetization of incompressible quantum spinon liquid were found.

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