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## 2D Majorana Flat Bands as Reason of Topological Superconductivity in Two-Dimensional $Z_2$ -Quantum Spin Liquid in $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$ Manganites

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As shown, the formation and destruction of 2D Majorana flat zones in frustrated  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  manganites occur as a result of Landau quantization of the spectrum of magnetic excitations of the  $Z_2$ -quantum spin liquid with magnetic flux in the form of composite quasi-particles ‘spinon-gauge field’. In this work, the dynamics of the formation and destruction of 2D Majorana flat zones in frustrated manganites is also studied by analysing of the dc field  $M(H)$  dependences measured in the zero-field-cooled (ZFC) and field-cooled (FC) measurement modes. As shown, in the processes of magnetization reversal of  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  at 4.2 K, a superposition of the hump-like asymmetric features of the  $M(H)$  plots is formed in the range of magnetic fields of  $\pm 500$  Oe, and non-dispersive ultra-narrow 2D Majorana flat zones of excited states of  $Z_2$ -chiral quantum spin liquid are formed in the range of weak magnetic fields of 100–200 Oe near the zero field, similarly to flat bands in the low-dimensional topological superconductors.

В даній роботі показано, що формування та руйнування 2D-надвузських Майоранових плоских зон у фрустрованих манганітах  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  відбуваються в результаті квантування Ландау спектру магнетних збуджень кіральної  $Z_2$ -квантової спінової рідини з магнетним потоком у вигляді композитних квазічастинок «спінон-калібрувальне поле». Цей результат добре узгоджується з результатами подібних досліджень топологічних надпровідників із 2D-пласкими зонами Майоранових ферміонів. Це дає змогу припустити, що можливою причиною локальної надпровідності у двовимірній  $Z_2$ -квантовій спіновій рідині в манганітах  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  є утворення надвузських зон Ландау в результаті квантування низькоенергетичного спектру Майоранових ферміонів калібрувальним полем.

**Key words:** 2D Majorana flat zones, Landau quantization,  $Z_2$ -chiral quantum spin liquid, gauge field of spinon-pairs’ spectrum, topological super-

conductors.

**Ключові слова:** двовимірні Майоранові пласкі зони, квантування Ландау,  $Z_2$ -хіральна квантова спінова рідина, калібрувальне поле спектру спінонних пар, топологічні надпровідники.

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## 1. INTRODUCTION

According to [1], in flat-band superconductors, the group velocity  $v_F$  of charge carriers is extremely small that leads to freezing of the kinetic energy. Superconductivity in this case seems impossible, since, within the framework of the BCS theory, this means the disappearance of such microproperties as the coherence length of Cooper pairs, their superfluidity rigidity, and the critical current. The authors reported the existence of a group velocity of free charge carriers in the two-layer graphene studied by them, which is characteristic of a graphene superlattice with a Dirac superconducting flat zone [2–6]. For the filling factor of the moiré superlattice in superconducting graphene  $1/2 < \nu < 3/4$ , a very small value of the group velocity  $v_F \sim 1000$  m/sec was found. It is important to note that the measurement of superfluidity, which controls the electrodynamic response of a superconductor, shows that it is dominated not by kinetic energy, but by an interaction-controlled superconducting gap, which is consistent with the theories of the quantum geometric contribution [2–6]. Evidence has been found for the crossover of electron-pairs' characteristic of the BCS and Bose–Einstein condensations [7–9].

According to [10], superconductivity is traditionally considered as a low-temperature phenomenon. Within the framework of the BCS theory, this is understood because of the fact that electron pairing occurs only near the usually two-dimensional Fermi surface, which is at a finite chemical potential. Because of this, the critical temperature is exponentially low as compared with microscopic scales of energy. On the other hand, pairing of electrons around a dispersionless (flat) energy band leads to very strong superconductivity with a critical temperature that depends linearly on the microscopic coupling constant. Flat bands can usually be generated only at surfaces and interfaces where high-temperature superconductivity can occur. The flat-gap character and low dimension also mean that, despite the high critical temperature, such a superconducting state will be subject to strong fluctuations. In [11], the superconducting properties of a two-dimensional Dirac material, such as deformed graphene, which in normal state has a spectrum of free

charge carriers with a flat energy band, were considered. It is shown that, in the superconducting state, the appearance of a flat energy band of carriers caused by deformation leads to a strong increase in the critical temperature of superconductivity compared to the case without deformation, an inhomogeneous order parameter with a two-peak shape of the local density of states, and a large, almost uniform and isotropic supercurrent.

According to [12], in systems with a condensed state, when a quasi-particle is a superposition of the electron and hole excitations and its production operator  $\gamma^\dagger$  becomes identical to the annihilation operator  $\gamma$ , such a particle can be identified as a Majorana fermion. Within the Reed–Green model, the Bogolyubov quasi-particles in the bulk become dispersive Majorana fermions, and the bound state formed in the core of the vortex becomes the Majorana zero mode. The former is interesting as a new type of wandering quasi-particles, while the latter is useful as a qubit for topological quantum computing. In condensed matter, the constituent fermions are electrons. Because the electron has a negative charge, it cannot be a Majorana fermion. Nevertheless, Majorana fermions can exist as collective excitations of electrons. The resulting Majorana fermions do not retain the true Lorentz invariance of the Dirac equation, since they do not move at the speed of light. However, with proper length and time scaling, the resulting Majorana fermions also obey the Dirac equation.

Such Majorana fermions appear within the boundaries of topological superconductors or in the class of spin-liquid systems. The condensation of bosons in the form of a bound state of Majorana fermions was previously studied in topological superconductors by tunnelling spectroscopy. The tunnelling conductivity spectra of topological superconductors depend on their size and symmetry. In one-dimensional topological superconductors with time-reversal violation, there is an isolated single Majorana zero mode at each end. Tunnelling conductance due to the isolated zero mode shows a differential conductance peak  $dI/dV$  with zero offset height  $2e^2/h$  [13–15]. If one Majorana zero mode is coupled to another Majorana zero mode at the other end of the superconductor, the tunnelling conductance is highly dependent on the coupling  $t$  between the Majorana modes at the different ends. When the ratio  $t/\Gamma$  of the coupling between modes to the width of the fermion spectrum  $\Gamma$  is very small, the peak shape  $dI/dV$  is realized [15]. However, in the case of significant mixing of the two Majorana modes, the differential conductivity has the form of a trough. In this case, the zero-bias conductance vanishes.

The cause of the coexistence of superconductivity and strong correlations in electron systems with flat bands was studied in [16].

Flat-band systems with a low density of charge-carrier states play an important role because the flat band-energy range is so narrow that the Coulomb interactions between free carriers  $E_c \propto e^2/a$  dominate over the kinetic energy, which puts these materials in a regime with strong correlations. If the flat band is narrow in both energy and momentum, its occupation can be easily changed in a wide range from zero to full. Landau levels are a striking example of two-dimensional flat bands. They occur when a strong magnetic field acts on a  $2D$  electron system. In this case, the electron motion is reduced by the Lorentz force to quantized cyclotron orbits. In this case, the translational degeneracy leads to completely flat zones, the width of which is completely determined by the degree of disorder. The partially-filled Landau levels, first studied in semi-conductor heterostructures, contain a rich set of competing orders, including ferromagnetism, charge-ordered band and bubble phases, and the best-known fractional quantum Hall liquids. The essential feature of these states is their intrinsic Berry curvature, which underlies their topological character and leads to integer and fractional quantum Hall effects. Twisted bilayer graphene has recently been found to exhibit highly correlated states and superconductivity. Thus, the formation of flat bands of Majorana fermions is a characteristic mechanism of topological superconductivity, the BCS and Bose–Einstein condensations of bosons in the form of a bound state of  $2D$  Majorana fermions and Dirac superconducting flat zone. The crossover of electron-pairs’ characteristic of the BCS and Bose–Einstein condensations of bosons in the form of a bound state of Majorana fermions (Majorana zero mode) was also studied in topological superconductors.

The phase transition of the quantum spin liquid (QSL) to a chiral state in  $2D$  frustrated antiferromagnets (AFMs) with different types of crystal lattice, caused by an external magnetic field close to  $H = 0$  Oe, has attracted great interest among theoreticians and experimenters [17–32]. It was shown that the transition of the QSL to the chiral state induced by an external magnetic field is accompanied by a phase transition into a phase with a topological order and excitation of fractional fermions (Majorana fermions). Kitaev was the first who constructed a quantitative model of the so-called  $Z_2$ -quantum spin liquid (a spin liquid with a local  $Z_2$ -magnetic flux in the unit cell) for spins  $S = 1/2$  located at the nodes of a quasi-two-dimensional hexagonal lattice [17]. The Kitaev’s Hamiltonian of the QSL describes the states of both gapped and gapless quantum spin liquids, which correspond to low-energy fractional excitations. According to Kitaev’s model, there is a strong anisotropic exchange between spins in the nearest neighbourhood of sites in a simple Ising form, but, because different bonds use different spin components, the ground magnetic state of the system becomes highly frustrated. According to the mod-

el, the ground state of such a system of spins in a zero magnetic field is a gapless quantum spin liquid, which can pass into a gap topological phase because of the action of perturbations that breaks the time reversal symmetry. One of such perturbations can be an external magnetic field perpendicular to the hexagonal layer. In Kitaev's theory, the phase transition of the QSL to the gap state is accompanied by the excitation of Majorana fermions (MF).

In elementary particle physics, a Majorana fermion is a particle that coincides with its antiparticle. In condensed matter physics, the concept of a Majorana fermion changes slightly: a Majorana fermion is a quasi-particle whose creation operator coincides with its annihilation operator. Interest in such quasi-particles is because they can theoretically be used in qubits for a topological quantum computer, while, due to their nonlocal nature, they are less sensitive to the influence of the environment. In one-dimensional systems, one speaks not of Majorana fermions, but of Majorana bound states, which do not move freely in the system, due to which they retain their properties. The possible experimental detection of such objects in combined semi-conductor–superconductor nanosystems in a magnetic field requires independent confirmation due to the complexity of detection and the existence of possible alternative explanations. The Hamiltonian used by Kitaev is very simple, since it contains exchange between nearest neighbours.

In this paper, results of study of spin (QSL) and superconducting (SC) quantum liquids in  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  are considered, which indicate the decisive role of flat Majorana-fermion bands' formation in this compound for the existence of these liquids and quantum phase transitions caused by external influences.

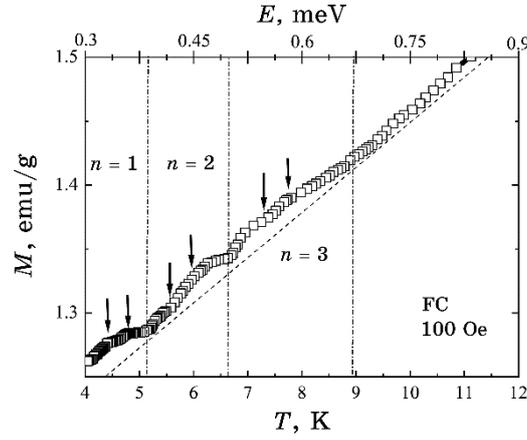
## 2. EXPERIMENTAL TECHNIQUE

Samples of self-doped  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  manganites ( $\delta \cong 0.1$ ) were obtained from high-purity oxides of lanthanum, samarium and electrolytic manganese taken in a stoichiometric ratio. The synthesized powder was pressed under pressure of 10 kbar into discs of 6 mm in diameter, of 1.2 mm in thick and sintered in air at a temperature of 1170°C for 20 h followed by a decrease in a temperature at a rate of 70°C/h. The resulting tablets were single-phase ceramic according to x-ray data. X-ray studies were carried out at 300 K on DRON-1.5 diffractometer in  $\text{NiK}_{\alpha_1+\alpha_2}$  radiation. Symmetry and parameters of the crystal gratings were determined by the position and character splitting reflections of the pseudo-cubic perovskite-type lattice. Temperature and field dependences of dc magnetization were obtained in the ZFC- and FC-measurement modes in the range of fields  $-5 \text{ kOe} \leq H \leq 5 \text{ kOe}$  at 4.2 K using a non-industrial magne-

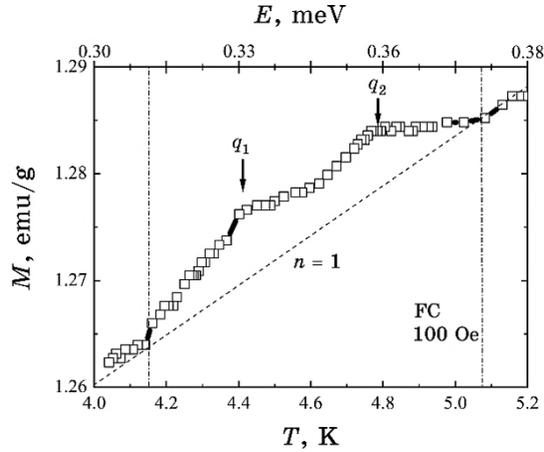
tometer.

### 3. EXPERIMENTAL RESULTS AND DISCUSSION

The evolution of the Landau quantization by a gauge field of the spinon-pairs' spectrum in frustrated  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  manganites with



**Fig. 1.** Quantum oscillations of the temperature dependence of the magnetization of  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  measured in a magnetic field  $H = 100$  Oe in the temperature range 4.2–12 K.



**Fig. 2.** Symmetrical doublet consisting of two overlapping peaks of the temperature dependence of the  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  magnetization measured in the Landau zone with number  $n = 1$  in a magnetic field  $H = 100$  Oe in the temperature range 4.0–5.2 K.

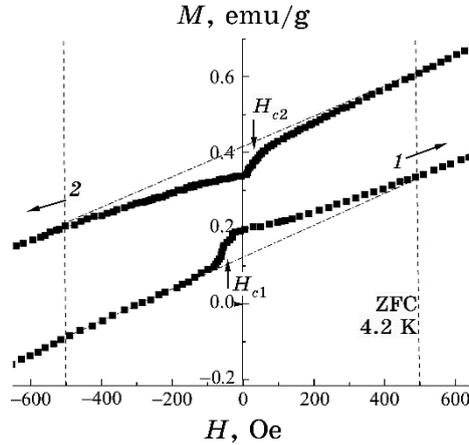
spinon Fermi surface was first researched in [33]. In the temperature range 4.2–12 K, the quantization of the spectrum of pairs of low-energy magnetic excitations  $Z_2$  of a quantum spin liquid with magnetic flux in the form of composite quasi-particles of the spinon–gauge field was found in a weak Mott insulator  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  (Figs. 1, 2). In magnetic fields with  $H = 100$  Oe, 350 Oe, 1 kOe, the spectrum has view of three narrow zones consisting of overlapping dual peak features of the magnetization equal to intensity near equidistant temperatures.

This made it possible to separate the detected continuous thermal excitations of magnetization into narrow overlapping zones  $n = 1$ ,  $n = 2$  and  $n = 3$  with a width of  $\cong 0.08\text{--}0.24$  meV. Formation of the continuous excitation spectrum of a quantum spin liquid in the regime of ‘weak magnetic fields’ with  $H = 100$  Oe, 350 Oe, 1 kOe is explained in terms of the Landau quantization models for the spectrum of composite quasi-particles with fractional values of the filling factor  $\nu$  for three overlapping Landau zones. In the regime of a ‘strong external magnetic field’ with  $H = 3.5$  kOe, new quantum oscillations of the temperature dependences of the magnetization of an incompressible liquid of spinons were found in the form of three narrow steps (plateaus) corresponding to the complete filling of non-overlapping Landau bands with integer values of the filling factor. The results of our study of the temperature dependences of  $M(T)$  for  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  are in a qualitative agreement with the results of the Balents study of Landau zones and superconductivity in moiré flat bands [16].

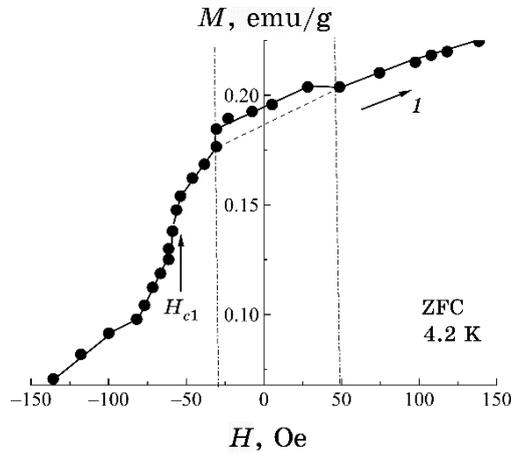
In this work, we have also studied the dynamics of the formation and destruction of 2D Majorana flat zones in frustrated manganites by analysing of the samples’ dc field dependences  $M(H)$  measured in the ZFC- and FC-measurement modes (Figs. 3–8). In the processes of magnetization reversal of  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  at 4.2 K in the ZFC- and FC-measurement modes, the sample magnetization  $M(H)$  grows linearly with increasing magnetic-field strength in a wide range of magnetic field  $\pm 5$  kOe, which is typical for paramagnets and anti-ferromagnets. However, the dc magnetization has clearly pronounced unusual features near zero field caused by the appearance of an additional supermagnetization during the transition of the QSL to a phase with a topological order. The strong field hysteresis is also unusual, the width of which in the FC-measurement mode is much larger than in the ZFC-mode. As can be seen in Fig. 3, in the process of magnetization reversal in the ZFC-mode near the critical fields  $H_{c1} \approx -50$  Oe and  $H_{c2} \approx 50$  Oe, thresholds’ features are formed for the magnetization isotherm 1 and demagnetization 2, respectively, followed by a smooth decrease in the supermagnetization of the sample to zero. Figure 3 clearly shows how the hump-like

asymmetric feature of the  $M(H)$  plots is formed in the range of magnetic fields of  $\pm 500$  Oe with increasing magnetic-field strength in the positive direction.

It is clearly seen that this unusual feature is a superposition of a relatively narrow threshold feature of magnetization near the criti-



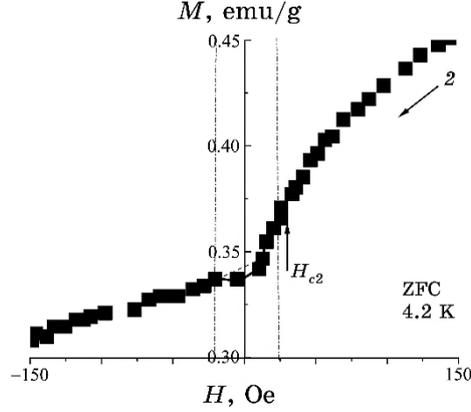
**Fig. 3.** Formation of the features of  $M(H)$  plots for  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  in ZFC-measurement mode at 4.2 K in the range of magnetic fields  $\pm 500$  Oe with an increase in the magnetic-field strength in the positive direction (isotherm 1) and in the negative direction (isotherm 2).



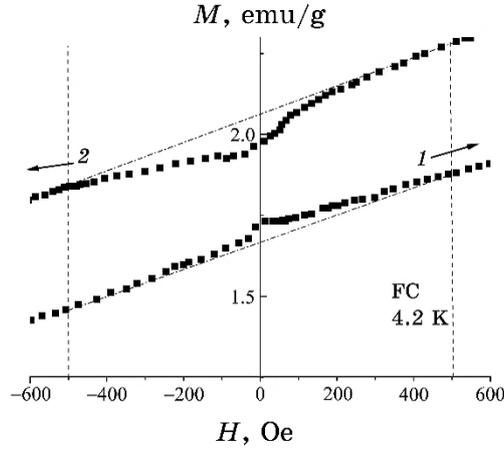
**Fig. 4.** Fragment of the field-dependent magnetization isotherm 1  $M(H)$  for  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  in the ZFC-measurement mode in the range of magnetic fields of  $\pm 150$  Oe. Formation of the hump-like and plateau-like features of the  $M(H)$  plots.

cal field  $H_{c1} \approx -50$  Oe of the phase transition of the QSL to the state with topological order, which is superimposed on a smooth drop in the supermagnetization to zero near magnetic fields of  $\pm 500$  Oe in the form of a tail.

Figure 3 shows that the dominant contribution to the growth of supermagnetization in isotherm 1 with increasing field is made by a



**Fig. 5.** Fragment of the field-dependent demagnetization isotherm 2  $M(H)$  for  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  in the ZFC-measurement mode in the range of magnetic fields of  $\pm 150$  Oe. Formation of a through-like feature of the  $M(H)$  plots.



**Fig. 6.** Formation of ultra-narrow plateau-like feature of the  $M(H)$  plots in FC-measurement mode for  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  in the range of magnetic fields of  $\pm 600$  Oe with an increase in the magnetic-field strength in the positive direction (isotherm 1) and wide through-like feature in the negative direction (isotherm 2).

clearly pronounced positive jump in magnetization near the field  $H_{c1} \neq 0$  Oe. These features are characteristic for the low-energy excitation of a wide 2D continuum of Majorana fermions during the phase transition of the chiral QSL to a state with a topological order, which was considered earlier in [17–32].

However, according to Fig. 4, there is also an additional contribution to the magnetization in the form of a plateau-like feature located in a narrow range of fields near the zero magnetic field.

This additional new feature is shaped like a hill with a cut top. Thus, the asymmetric feature of the  $M(H)$  graphs obtained by us in the range of magnetic fields of  $\pm 500$  Oe in the ZFC-mode with increasing magnetic-field strength in the positive direction (isotherm 1) consists of two contributions, which are different in shape and intensity: asymmetric hump-like feature in the range of fields of  $\pm 500$  Oe and narrower plateau-like feature near  $H = 0$  Oe.

A new complex feature of the  $M(H)$  plots is formed with decreasing field during reversal cycle, shown on isotherm 2 in Fig. 5.

Figure 5 clearly shows that the magnetic response of the spin system during sample demagnetization near zero field has the form of a threshold drop in supermagnetization with decreasing field near the critical value  $H_{c2} \approx 50$  Oe and unusual additional a through-like feature near  $H = 0$  Oe. This diamagnetic drop of magnetization may be connected with excitation of the coupled Majorana zero modes in one-dimensional topological superconductor with time reversal violation [13–15].

Critical fields, shape and intensity of a threshold low-energy excitation of quasi-particles to the total magnetization on two isotherms differ significantly, *i.e.*, there is a strong hysteresis in the processes of magnetization–demagnetization of the sample. The unusual asymmetry of the  $M(H)$  curves for the left ( $H < 0$  Oe) and right ( $H > 0$  Oe) halves of the field dependences of the QSL magnetization in isotherms 1 and 2 should also be noted that indicates the violation of the mirror symmetry of the sample magnetization upon reversal of the sign of the external magnetic field. The magnetization reversal isotherms 1, 2 obtained in the FC-measurement mode are also of great interest (see Figs. 6–8, in which it can be seen how plateau-like and through-like features of the magnetization are formed during the magnetization reversal of the sample near zero magnetic field). As can be seen in Fig. 6, during the magnetization reversal of  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  at 4.2 K, relatively narrow plateau-like features of isotherms 1 and 2 with different widths and intensities occur near  $H = 0$  Oe. These features of supermagnetization, which arise when the direction of the vector  $\mathbf{H}$  is inverted, are superimposed on a smooth drop in magnetization to zero with increasing field strength near magnetic fields of  $\pm 500$  Oe.

Figure 7 shows that, during sample magnetization with increasing  $H$  (isotherm 1) near  $H = 0$  Oe, there is a jump of magnetization as a very narrow plateau-like feature of  $M(H)$  curves in the form of cut top of a hill. More clearly, the formation of additional feature of supermagnetization near zero magnetic field manifests itself in isotherm 2 during the demagnetization of the sample in the form of a

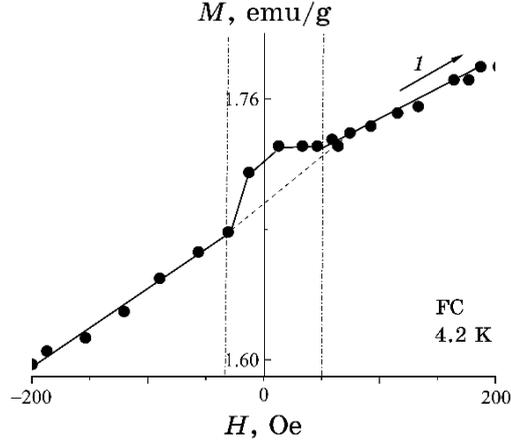


Fig. 7. Fragment of the field-dependent magnetization isotherm 1  $M(H)$  for  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  in the FC-measurement mode in the range of magnetic fields of  $\pm 200$  Oe. Formation of an ultra-narrow plateau-like feature of the  $M(H)$  plots.

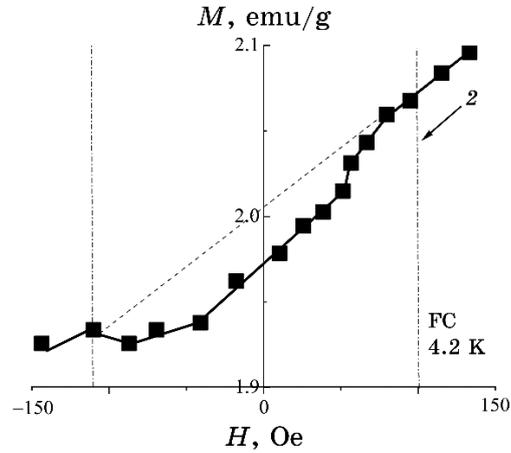


Fig. 8. Fragment of the field-dependent demagnetization isotherm 2  $M(H)$  for  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  in the FC-measurement mode in the range of magnetic fields of  $\pm 150$  Oe.

wide trough-like drop in magnetization near  $H = 0$  Oe (Fig. 8). Thus, in the processes of magnetization reversal of  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  at 4.2 K in the ZFC- and FC-measurement modes in the range of weak magnetic fields of 100–200 Oe near the zero field, a non-dispersive ultra-narrow zone of excited states of the  $Z_2$ -chiral QSL is formed similarly to flat bands in topological superconductors. We interpret these low-energy excitations of  $2D$  Majorana fermions as the result of the Landau quantization of low-energy magnetic excitations of the  $Z_2$ -chiral QSL by a gauge field in the form of composite quasi-particles of the spinon–gauge field with increasing temperature and external magnetic-field strength.

According to [30], the broad hump-like response of neutron scattering by a system of spins in the QSL has several different causes. First, this is scattering, in which any movement of spins with a wave vector  $\mathbf{q} = \mathbf{0}$  requires the creation of many magnetic excitations. Secondly, this is not a simple excitation of magnons, since only fractional magnetic excitations participate in it, which are created simultaneously, and, thus, the scattering process is multiparticle. Third, when describing the low-energy spectrum, quasi-particles are not in the first place; so, in principle, the dispersion relation  $\omega = \omega(\mathbf{q})$  is insignificant. In the case of the  $Z_2$ -spin liquid, in which the triplet excitation decays into a pair of  $S = 1/2$  spinons, fractional magnetic excitations are considered as magnetic moments within the framework of the quantum dimer model. In order to be observable, each individual spinon must behave like a coherent free particle in the limit of low excitation energies. There is a possibility of quantum chirality deconfinement at the bicritical point of the phase diagram, which separates two phases with the presence of quantum spin liquid excitations' confinement [31, 32].

The deconfinement of bound states of fractional particles is accompanied by the expenditure of energy required for their decoupling. This does not exclude the possibility of formation of new discrete composite states with a finite binding energy. One such case is crushing with a very large asymmetry of excitation, in which the flow can be very heavy and slow-moving, *i.e.*, has a very small zone width. Thus, it can interact with an arbitrary number of moments with almost constant energy and, thereby, produce the conservation of extremely inefficient moments. This leads to broad features of particle excitation, which are typical for photons. The emergence of Majorana fermions is the most prominent characteristic of topological superconductors. The electron and hole excitations are superimposed in the superconducting state; so, they become indistinguishable. This makes a superconductor to obtain particle–hole symmetry, with which the topological gapless boundary excitations become Majorana fermions. Hence, if the vortices obey the conventional Bose

or Fermi statistics, the final state remains the same as the initial one. However, in a 2D chiral  $p$ -wave superconductor, due to the existence of the Majorana zero mode, the wave function of the final state is different from that of the initial state. A vortex in a superconductor contains a flux quantum  $\Phi_0 = hc/(2e)$ ; therefore, when an electron or a hole moves around a vortex, the state acquires the same phase  $-1$  due to the Aharonov–Bohm effect. Consequently, a quasi-particle in the superconductor, which is a superposition of an electron and a hole, acquires the same factor  $-1$ . This means that vortices with Majorana zero modes are neither bosons nor fermions. An isolated Majorana zero mode also appears at each end point of a 1D topological superconductor [36]. In general, as discussed by Ivanov [38], an exchange of the Majorana zero modes  $\gamma_0^{(i)}, \gamma_0^{(j)}$  is represented by the unitary transformation. One can exchange such Majorana zero modes in a network of 1D topological superconductors [38–48]. Again, the exchange operators of those Majorana zero modes are given by  $U_{ij}$ , and the Majorana zero mode at the end point is also a non-Abelian anyon [44, 48]. The non-Abelian anyon is expected to have an application in quantum computing [49, 50].

#### 4. CONCLUSION

According to the experimental results presented in this paper, the formation and destruction of 2D ultra-narrow Majorana flat bands in frustrated  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  manganites occur as a consequence of Landau quantization of the magnetic-excitation spectrum of a chiral  $Z_2$ -quantum spin liquid with a magnetic flux in the form of composite quasi-particles ‘spinon–gauge field’. These results of our study of the temperature and field dependences of the supermagnetization features for  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  are in well qualitative agreement with the results of similar researches in the low-dimensional flat-band topological superconductors. This allows us to suggest that a possible reason for local superconductivity in two-dimensional  $Z_2$ -quantum spin liquid in  $\text{La}_{0.15}\text{Sm}_{0.85}\text{MnO}_{3+\delta}$  manganites is the formation of ultra-narrow Landau zones because of quantization of the low-energy spectrum of Majorana fermions by a gauge field.

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